POTENTIAL & CONCEPT EDUCATIONS
NEET | AIIMS JEE | KVPY | OLYMPIAD
JOINING HANDS FOR A BETTER TOMORROW

SOLUTIONS

WITH

ANSWER KEY

ANTS-FT # 02

DROPPER ENGINEERING

(PHYSCIS, CHEMISTRY & MATHS)

TARGET : JEE (MAIN + ADVANCED) 2019-20

Exam. Date : 29-12-2019

POTENTIAL & CONCEPT EDUCATIONS

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JOINING HANDS FOR A BETTER TOMORROW

ANSWER KEYS FOR ANTS-FT # 02 (TARGET – JEE-MAIN-2020)

DATE : 29-12-2019

ANSWERS [PHYSICS]

1. C	2. C	3. A	4. D	5. B	6. D	7. B	8. A	9. B	10. B
11. D	12. B	13. D	14. A	15. B	16. C	17. B	18. D	19. D	20. D
21. (2)	22. (3)	23. (5)	24. (4)	25. (8)					

ANSWERS [CHEMISTRY]

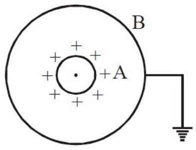
26. C	27. D	28. B	29. D	30. D	31. A	32. A	33. A	34. B	35. A
36. B	37. A	38. B	39. C	40. B	41. B	42. C	43. B	44. B	45. B
46. (12)	47. (150)		48. (6)	49. (3)	50. (4)				

ANSWERS [MATHS]

51. B	52. B	53. D	54. C	55. C	56. C	57. A	58. B	59. D	60. C
61. A	62. C	63. B	64. D	65. B	66. B	67. A	68. C	69. A	70. B
71. (2)	72. (2)	73. (7)	74. (1)	75. (7)					

❒❒❒❒❒❒❒

1. A and B are concentric conducting spherical shells. A is given a positive charge while B is earthed. Then:-



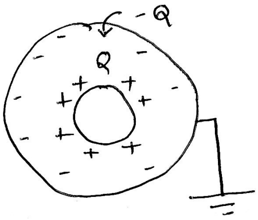
(A) A and B both will have the same charge densities

(B) The potential inside A and outside B will zero.

(C) the electric field between A and B is non zero

(D) the electric field inside A and outside B is non zero.

Solution :



To make potential zero at outer surface charge zero at outer surface but inner charge same with different charge density.

→ Potential due to both charges are opposite but unequal inside small sphere so not zero.

→ Electric field in between A & B.

Hence the answer is (C).

2. A body takes 10 minutes to cool down from 62°C to 50°C. If the temperature of surrounding is 26°C then in the next 10 minutes temperature of the body will be :-

(A) 38°C (B) 40°C (C) 42°C (D) 44°C

Solution :

$$\ln\left[\frac{T_1 - T_0}{T_2 - T_0}\right] = Kt$$

$$\Rightarrow \ln\left(\frac{62-26}{50-26}\right) = K(10) \Rightarrow \ln\left(\frac{36}{24}\right) = K(10) \quad \text{—(1)}$$

$$\Rightarrow \ln\left(\frac{50-26}{\theta-26}\right) = K(10) \Rightarrow \ln\left(\frac{24}{\theta-26}\right) = K(10) \quad \text{—(2)}$$

equate $\Rightarrow \dfrac{36}{24} = \dfrac{24}{\theta-24} \Rightarrow 36\theta - 26\times 36 = 576$

$\Rightarrow \theta = 42°C$

Hence the answer is (C).

3. After charging a capacitor the battery is removed. Now by placing a dielectric slab between the plates :-

(A) The potential difference between the plates and the energy stored will decrease but the charge on plates will remain same

(B) the charge on the plates will decrease and the potential difference between the plates will increase

(C) the potential difference between the plates will increase and energy stored will decrease but the charge on the plates will remain same

(D) the potential difference, energy stored and the charge will remain unchanged.

Solution :

$C' = KC \Rightarrow$ charge will unaffected, $Q' = Q$

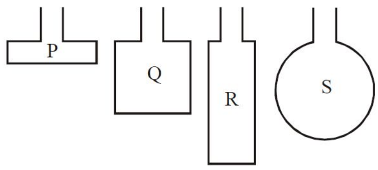
$C'V' = CV \Rightarrow V' = V/K \rightarrow$ decreases

$E = \frac{1}{2}CV^2 = \frac{1}{2}QV \Rightarrow E' = \frac{1}{2}QV'$

$E' < E \Rightarrow$ decreases.

Hence the answer is (A).

4. Four wires of equal length are bent in the form of four loops P, Q, R and S. These are suspended in a uniform magnetic field and same current is passed in them. The maximum torque will act on :-



(A) P (B) Q (C) R (D) S

Solution :

$\vec{\tau} = \vec{M} \times \vec{B}$ For perpendicular $\tau = MB$

$M = IA$ So τ max. if area maximum

for given perimeter area of circle is maximum.

Hence the answer is (D).

5. The wavelength of L_α line in X-ray spectrum of $_{78}Pt$ is 1.32 Å. The wavelength of L_α line in X-ray spectrum of another unknown element is 4.17 Å. If screening constant for L_α line is 7.4, then atomic number of the unknown

element is:

(A) 78 (B) 47 (C) 40 (D) 35

Solution :

$\sqrt{\nu} = a(z-b)$, $\nu = \frac{c}{\lambda}$

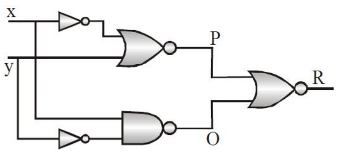
$\left(\frac{c}{1.32}\right)^{1/2} = a(78-7.4)$ —① $\left(\frac{c}{4.17}\right)^{1/2} = a(z-7.4)$ —②

divide

$(3.6)^{1/2} = \frac{70.6}{z-7.4}$, $z = 47$ approx.

Hence the answer is (B).

6. Figure gives a system of logic gates. From the study of truth table it can be found that to produce a high output (1) at R, we must have



(A) X = 0, Y = 1 (B) X = 1, Y = 1 (C) X = 1, Y = 0 (D) Not possible

Solution :

$R = \overline{\overline{\bar{x}+y} + \overline{x\bar{y}}}$

$= (\bar{x}+y)(x\bar{y}) = 0$, R = 1 Not possible.

Hence the answer is (D).

7. An electron microscope is operated at 40 kV. The ratio of resolving power of this microscope and another one which uses yellow light of wavelength 6×10^{-7} m, is :-

(A) 9.78×10^{6} (B) 9.78×10^{4} (C) 9.78×10^{-4} (D) 9.78×10^{-6}

Solution :

Resolving power $R \propto \frac{1}{\lambda}$

$\lambda_1 = \frac{1.227}{\sqrt{V}} = \frac{1.227}{\sqrt{4\times10^4}}\, nm = \frac{1.227}{2\times10^2}\times10^{-9}$

$\lambda_2 = 6\times10^{-7}\, m$ $= 0.613\times10^{-11}\, m$

$\frac{R_1}{R_2} = \frac{\lambda_2}{\lambda_1} = \frac{6\times10^{-7}}{0.613\times10^{-11}} = 9.78\times10^{4}$

Hence the answer is (B).

8. In an adiabatic expansion the product of pressure and volume.

(A) decreases (B) increases

(C) remains constant (D) first increases, then decreases.

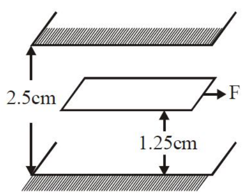
Solution :

PV^{r} = Constant $\Rightarrow PV^{r-1}V$ = Constant

$\Rightarrow PV = \frac{\text{Constant}}{V^{r-1}}$ as $V\uparrow, (PV)\downarrow$

Hence the answer is (A).

9. A space 2.5 cm wide between two large plane surfaces is filled with oil. Force required to drag a very thin plate of area 0.5m^2 just midway the surfaces at a speed of 0.5 m/sec is 1 N. The coefficient of viscosity in kg-s/ m^2 is :



(A) 5×10^{-2} (B) 2.5×10^{-2} (C) 1×10^{-2} (D) 7.5×10^{-2}

Solution :

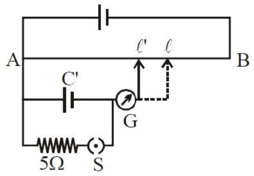
$$F = -\eta A \frac{\Delta v}{\Delta x}$$

$$\Rightarrow 1 = \eta\, 2\times0.5\times\frac{0.5\times100}{1.25} \Rightarrow \eta = 2.5\times10^{-2}\ kg\text{-}s/m^2$$

Here area double beceuse side are both or same force up and down.

Hence the answer is (B).

10. In the potentiometer circuit as shown in the figure, the balance length Aℓ = 60 cm when switch S is open. When switch S is closed and the value of R is 5Ω, the balance length Aℓ' = 50 cm. The internal resistance of the cell C' is:



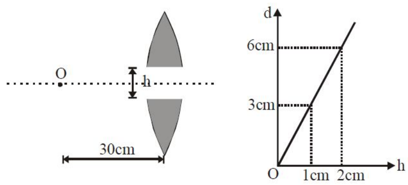
(A) 1.2 Ω (B) 1.0 Ω (C) 0.8 Ω (D) 0.6 Ω

Solution :

$$r = \frac{l_1 - l_2}{l_2}\times R \Rightarrow r = \frac{60-50}{50}\times 5 \Rightarrow r = 1\,\Omega$$

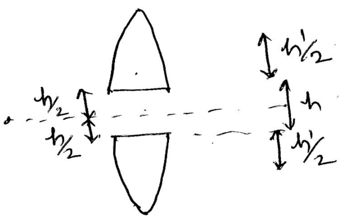
Hence the answer is (B).

11. Figures shows a convex lens cut symmetrically into two equal halves and separated laterally by a distance h. A point object placed at a distance 30 cm from the lens halves, forms two real images separated by a distance d. A plot of d versus h is shown in figure. The focal length of the lens is :-



(A) 60 cm (B) 40 cm (C) 45 cm (D) 20 cm

Solution :



h' = Image height from convex lens

$d = h' + h \Rightarrow 3 = h' + 1 \Rightarrow h' = 2\,cm$

$$\frac{h'}{h} = -2 = \frac{f}{u+f}$$

$\Rightarrow \underline{f = 20\,cm.}$

Hence the answer is (D).

12. Rising and setting sun appears to be reddish because :

(A) Diffraction sends red rays to the earth at these times

(B) Scattering due to dust particles and air molecules are responsible

(C) refraction is responsible

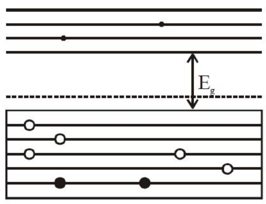
(D) polarization is responsible

Solution :

In morning and evening light travels more distance in atmosphere, shorter wavelengths of light like blue scattered by air or dust and light of longer wavelength reached to eyes, So it appears to be red.

Hence the answer is (B).

13. In the energy band diagram of a material shown in figure, the open circles and filled circles denote holes and electrons respectively. The material is :



(A) an insulator

(B) a metal

(C) a n-type semiconductor

(D) a p-type semiconductor

Solution :

In p-type of semiconductor holes are more and energy band of accepter is near to valance band.

Hence the answer is (D).

14. In the Young's double slit experiment, the intensities at two points P_1 and P_2 on the screen are respectively I_1 and I_2. If P_1 is located at the centre of a bright fringe and P_2 is located at a distance equal to a quarter of fringe width from P_1, then I_1 / I_2 is :

(A) 2

(B) 1/2

(C) 4

(D) 16

Solution :

$y = \frac{\beta}{4} = \frac{\lambda D}{4d} \Rightarrow \Delta x = \frac{yd}{D} = \frac{\lambda}{4}$ = Path difference

Phase difference $= \Delta\phi = \frac{\lambda}{4} \times \frac{2\pi}{\lambda} = \frac{\pi}{2}$

$I_2 = I_1 \cos^2\frac{\phi}{2} \Rightarrow I_2 = I_1 \times \frac{1}{2} \Rightarrow \frac{I_1}{I_2} = 2:1.$

Hence the answer is (A).

15. A block of mass m is placed on a smooth horizontal surface. A force making an angle θ with the horizontal starts acting on the block. The magnitude of the force is constant but its direction with the horizontal changes as θ = a+ bs, where a and b are constants and s is the distance covered by the block. If |F| = 2mb, find the velocity of the block as a function of the angle θ.

(A) $v = 4(\cos\theta + \cos a)^{1/2}$ (B) $v = 2(\sin\theta \,.\, \sin a)^{1/2}$

(C) $v = 4(\sin\theta \,.\, \sin a)^{1/2}$ (D) $v = 2(\cos\theta + \cos a)^{1/2}$

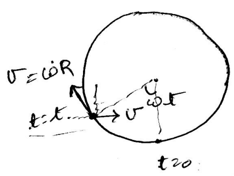
Solution :

Hence the answer is (B).

16. Consider a point P on the circumference of a disc rolling along a horizontal surface. If R is the radius of the disc, the distance through which P moves in one full rotation of the disc is:

(A) $2\pi R$ (B) $4\pi R$ (C) 8R (D) πR

Solution :



for 1 rotation

$\omega t = 2\pi$

$t = \frac{2\pi}{\omega}$

$$|\vec{v}| = \sqrt{v^2 + v^2 + 2v^2\cos(180^\circ - \omega t)}$$

$$= v\sqrt{2(1+\cos\omega t)}$$

$$= 2v \sin\frac{\omega t}{2}$$

$$\text{distance} = \int |\vec{v}|\, d(t)$$

$$= \int_0^{2\pi/\omega} 2v \sin\frac{\omega t}{2}\, d(t)$$

$$= 2v\left[-\frac{\cos\frac{\omega t}{2}}{(\omega/2)}\right]_0^{2\pi/\omega}$$

$$= \frac{4v}{\omega}[2] = \frac{8v}{\omega} = 8R$$

Hence the answer is (C).

17. The masses and radii of the earth and the moon are M_1, R_1 and M_2, R_2 respectively. Their centres are distance d apart. The minimum speed with which particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is :

(A) $v = \sqrt{\dfrac{4g(M_1+M_2)}{d}}$ (B) $v = \sqrt{\dfrac{4G(M_1+M_2)}{d}}$

(C) $v = \sqrt{4G(M_1+M_2)}$ (D) $v = \sqrt{4Gd(M_1+M_2)}$

Solution :

$$-\frac{GM_1m}{d/2} - \frac{GM_2m}{d/2} + \frac{1}{2}mv^2 = 0$$

$$v = \sqrt{\frac{4G(M_1+M_2)}{d}}$$

Hence the answer is (B).

18. A wooden block floats in a liquid with 40% of its volume inside the liquid. When the vessel containing the liquid starts rising upwards with acceleration a = g/2, the percentage of volume inside the liquid is :

(A) 20% (B) 60% (C) 30% (D) 40%

Solution :

Buoyancy force is equal to weight displaced
Here $W = \rho g' V$ both side has same
change.

Hence the answer is (D).

19. Two very long, straight, parallel wires carry steady currents I and .I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity v is perpendicular to the plane of wires. The magnitude of the force due to the magnetic field acting on the charge at this instant is:

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{2\mu_0 Iqv}{\pi d}$ (C) $\frac{\mu_0 Iqv}{\pi d}$ (D) 0

Solution :

Force due to both wire is zero because $\vec{v}$ and
$\vec{B}$ both are in same direction.

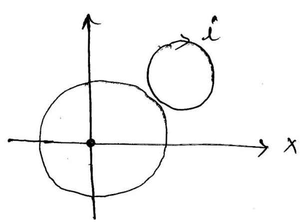
Hence the answer is (D).

20. A long straight wire is carrying current I_1 in +z direction. The x-y plane contains a closed circular loop carrying current I_2 and not encircling the straight wire. The force on the loop will be:

(A) $\mu_0 I_1 I_2 / 2\pi$ (B) $\mu_0 I_1 I_2 / 4\pi$

(C) 0 (D) depends on the distance of the centre of the loop from the wire.

Solution :

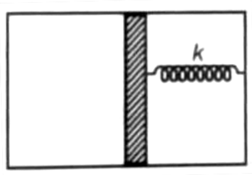


Thus the force depends on
the distance of the center
of the loop from the wire.

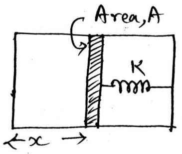
Hence the answer is (D).

Integer Type :

21. A thermally insulated vessel is divided into two parts by a heat insulating massless piston which can move in the vessel without friction. The left part of vessel contains 1 mole of an monoatomic gas and right part is empty. The piston is connected to the right wall of vessel through a spring whose length in free state is eual to the length of the vessel. Heat capacity of the system is found to be nR. Find n. Heat capacities of the vessel piston and spring are negligible.



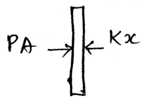
Solution :



$$dQ = dU + dW$$

$$1 \cdot C\,dT = \frac{3}{2} \cdot 1 \cdot R\,dT + Kx\,dx$$

At equilibrium



$$x = \frac{PA}{K}$$

$$\Rightarrow x^2 = \frac{PAx}{K}$$

$$\Rightarrow x^2 = \frac{PV}{K} = \frac{RT}{K}$$

$$\Rightarrow 2x\,dx = \frac{R\,dT}{K}$$

Putting value of $x\,dx$

$$1 \cdot C\,dT = \frac{3}{2} R\,dT + K \cdot \frac{R\,dT}{2K}$$

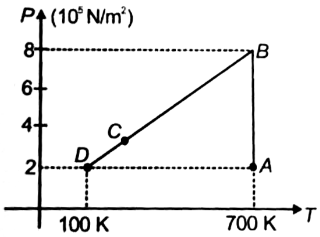
$$1 \cdot C\,dT = 2R\,dT$$

$$C = 2R$$

$$\boxed{n = 2}$$

Hence the answer is (2).

22. An ideal gas undergoes through a process as shown in the figure. At state C, $5V_C = V_A$. The pressure of the gas at C is $\frac{1}{n} \times 10^6 \,\text{N/m}^2$. Find the value of n



Solution :

Hence the answer is (3).

23. A physical quantity Q depends upon three quantities x, y and z as $Q = \frac{x^2 y^{1/2}}{z}$; in a particular set of measurements, x is measured with +50% error, y is measured with - 36% error and z is measured with - 20% error. The percentage error in the calculation of Q in this set of easurements is 25 x n%. Find the value of n.

Solution :

$$Q = \frac{x^2 y^{1/2}}{z}, \quad Q' = \frac{(x')^2 (y')^{1/2}}{z'}$$

$$= \frac{(1.5x)^2 (0.64y)^{1/2}}{(0.8z)}$$

$$= \frac{2.25 \times 0.8}{0.8} \frac{x^2 y^{1/2}}{z}$$

$$Q' = 2.25Q$$

$$\%\ \text{Error} = \frac{Q' - Q}{Q} \times 100$$

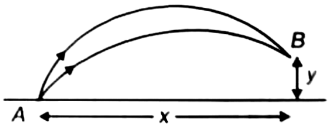
$$= 125$$

$$= 25 \times 5$$

$$\boxed{n = 5}$$

Hence the answer is (5).

24. A boy throws a ball with a speed $20\left(\sqrt{3}+1\right)$ m/s at angle 60^0 from horizontal. If he throws another ball with the same speed at an angle 30^0, determine the time interval between the two throws so the balls collide in mid air at B. (Take $g = 10 m/s^2$)



Solution :

Let 60' as ① and 30' as ②, 60' required more time to cover same horizontal distance

$t_1 = t + x, \; t_2 = t$

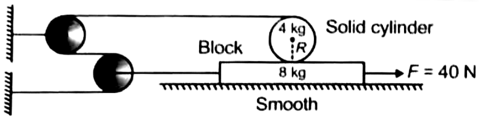
$x_1 = x_2 \Rightarrow u\frac{\sqrt{3}}{2}t = \frac{1}{2}u(t+x) \Rightarrow \boxed{x + t = \sqrt{3}t}$

$y_1 = y_2 \Rightarrow \frac{u}{2}t - 5t^2 = \frac{\sqrt{3}}{2}ut \times \sqrt{3} - 15t^2$

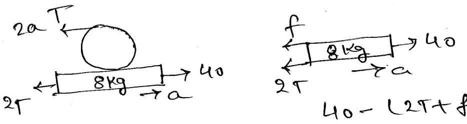
$\Rightarrow t = 2(\sqrt{3}+1), \; x = 4$ sec.

Hence the answer is (4).

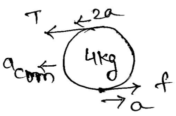
25. An ideal string is warpped several times on a solid cylinder of mas 4kg and radius 1m. The pulleys are ideal and the surface between block and ground is smooth. If the torque acting on the cylinder is $\frac{10x}{9} N-m$, then find the value of x.



Solution :



$40 - (2T + f) = 8a$ — ①



$a_{com} + \alpha R = 2a$

$a_{com} - \alpha R = -a$

$2a_{com} = a \Rightarrow a_{com} = a/2, \; \alpha = \frac{3a}{2R}$

$T - f = 4 \cdot a/2 = 2a \Rightarrow T - f = 2a$ — ②

$(T+f)R = I\alpha \Rightarrow T + f = 2 \cdot \frac{3a}{2R} \Rightarrow T + f = 3a$ — ③

$T = 2.5a, \; f = 0.5a$

From eq① $40 - (5a + 0.5a) = 8a$

$\Rightarrow 40 = 13.5a \Rightarrow \frac{80}{27} = a$

Torque $= 5.5a = \frac{3\cancel{11}}{2} \times \frac{80}{27} = \frac{80}{9}$

$\boxed{x = 8}$

Hence the answer is (8).

❒❒❒❒❒❒❒❒❒❒

26. Match the column-I with column-II

Column-I (Orbitals involved in the hybridization)	**Column-II** (Predicted Geometry)
(P) $s, p_x, p_y, p_z, d_{x^2-y^2}, d_{z^2}$	(1) Trigonal bipyramidal
(Q) $s, p_x, p_y, d_{x^2-y^2}$	(2) Tetrahedral
(R) s, P_x, P_y, P_z	(3) Square planar
(S) $s, p_x, p_y, p_z, d_{z^2}$	(4) Octahedral

	P	Q	R	S
(A)	1	2	3	4
(B)	1	4	2	4
(C)	4	3	2	1
(D)	4	2	1	3

Solution :

Hence the answer is (C).

27. In which process does the nitrogen undergo oxidation ?

(A) $N_2 \rightarrow 2NH_3$ (B) $N_2O_4 \rightarrow 2NO_2$ (C) $NO_3^- \rightarrow N_2O_5$ (D) $NO_2^- \rightarrow NO_3^-$

Solution :

$NO_2^- \rightarrow NO_3^-$

Hence the answer is (D).

28. $Ag_2S + NaCN + Zn \longrightarrow Ag$

This method of extraction of Ag by complex formation and then its displacement is called

(A) Parke's method (B) Mac Arthur-Forest method

(C) Serpek method (D) Hall's method

Solution :

Mac Arthur-Forest method

Hence the answer is (B).

29. ZnO shows yellow colour on heating due to

(A) d-d transition

(B) C-T spectra

(C) Higher polarization caused by Zn^{2+} ion

(D) F-centres

Solution :

F-centres

Basec on theory

Hence the answer is (D).

30. The pair of amphoteric hydroxide is

(A) $Al(OH)_3$, LiOH (B) $Be(OH)_2$, $Mg(OH)_2$

(C) $B(OH)_3$, $Be(OH)_2$ (D) $Be(OH)_2$, $Zn(OH)_2$

Solution :

$Be(OH)_2$, $Zn(OH)_2$

Hence the answer is (D).

31. The critical pressure P_C and critical temperature T_C for a gas obeying van der Waal's equation are 80 atm and 87°C. Molar mass of the gas is 130 g/mole. The compressibility factor for the above gas will be smaller than unity under the following condition

(A) 1 atm and 800°C (B) 1 atm and 1200°C

(C) 1 atm and 1000°C (D) 1 atm and 1100°C

Solution :

1 atm and 800°C

$T_C = 87°\ C$

$T_B = \frac{27}{8} \times 360 = 1215\ K$ or $942°\ C$

Hence Z < 1 at T < 942° C

Hence the answer is (A).

32. A brown ring is formed in the ring test for NO_3^- ion. It is due to the formation of

(A) $[Fe(H_2O)_5(NO)]^{2+}$ (B) $FeSO_4.NO_2$

(C) $[Fe(H_2O)_4(NO)_2]^{2+}$ (D) $FeSO_4.HNO_3$

Solution :

When freshly prepared solution of $FeSO_4$ is added in a solution containing NO_3^- ion, it leads to formation of a brown coloured complex. This is known as brown ring test of nitrate.

$NO_3^- + 3Fe^{2+} + 4H^+ \rightarrow NO + 3Fe^{3+} + 2H_2O$

$[Fe(H_2O)_6]^{2+} + NO \rightarrow \underset{\text{Brown ring}}{[Fe(H_2O)_5(NO)]^{2+}} + H_2O$

Hence the answer is (A).

33. When a small amount of solid calcium phosphide Ca_3P_2 is added to water, what are the most likely products ?

(A) Aqueous Ca^{2+} and OH^- ions and gaseous PH_3

(B) Aqueous Ca^{2+} and OH^- ions and gaseous H_3PO_3

(C) Solid CaH_2 and aqueous H_3PO_2

(D) Solid CaO and aqueous PH_3

Solution :

Aqueous Ca^{2+} and OH^- ions and gaseous PH_3

$Ca_3P_2 + 3H_2O \longrightarrow 3Ca(OH)_2 + 2PH_3\uparrow$

Hence the answer is (A).

34. The mineral beryl contains 5.03% beryllium by mass and contains three beryllium atoms per formula unit. Determine the formula mass of beryl. [Be = 9]

(A) 950 g/mol (B) 537 g/mol (C) 270 g/mol (D) 179 g/mol

Solution :

537 g/mol

$\frac{5.03}{100} \times M = 3 \times 9 \quad \Rightarrow \quad M = 537$

Hence the answer is (B).

35. Which of the following statements are correct ?

(1) The energy of light is inversely proportional to its wavelength

(2) Electrons behave as both waves and particles

(3) The typical atom can emit only certain types of energy is excited

(4) Infrared radiations have higher energy than gamma rays

(A) 1, 2 and 3 (B) 1 and 3 (C) 2 and 4 (D) Only 4

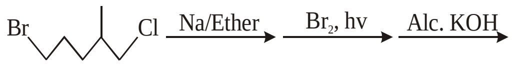
Solution :

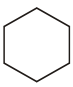
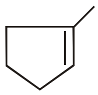
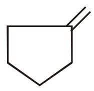
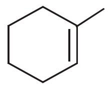
1, 2 and 3

Gamma radiation is more energetic than IR radiation.

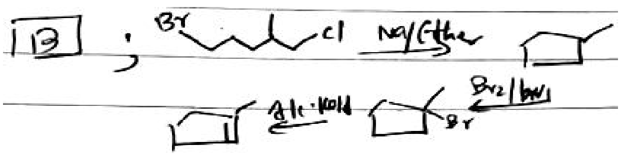
Hence the answer is (A).

36. The major product of the following reaction is :

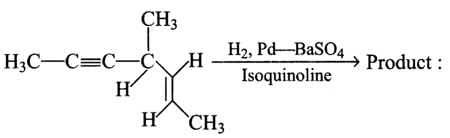


(A)  (B)  (C)  (D) 

Solution :



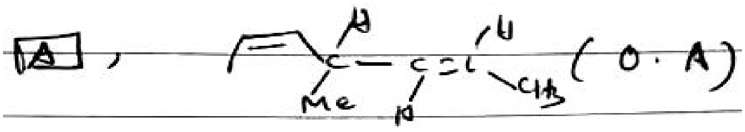
Hence the answer is (B).

37. 

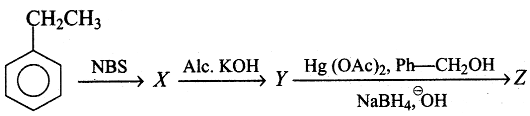
(A) an optically active compound (B) an optically inactive compound

(C) a racemic mixture (D) a diastereomeric mixture

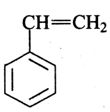
Solution :

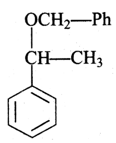


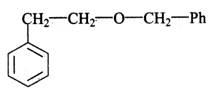
Hence the answer is (A).

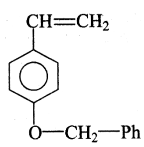
38. 

Z; 'Z' is :

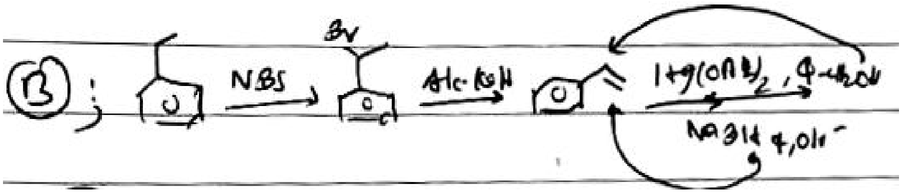
(A) 

(B) 

(C) 

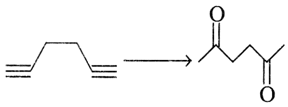
(D) 

Solution :



Hence the answer is (B).

39. How is the following trasformation best carried out ?

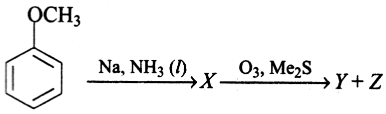


(A) OsO_4; $NaHSO_3$ (B) H_2SO_4/H_2O (C) $HgSO_4/H_2SO_4$ (D) HIO_4

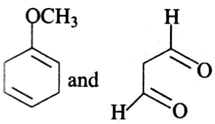
Solution :

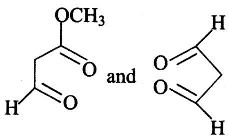
(C) ; Kucherov Reaction [Follow M.K Addition

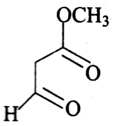
Hence the answer is (C).

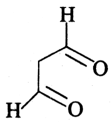
40. 

Identify products Y and Z.

(A) 

(B) 

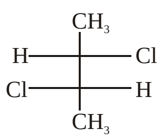
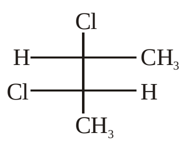
(C) Both are 

(D) Both are 

Solution :

[B] ; First Birch Red. then ozonolysis

Hence the answer is (B).

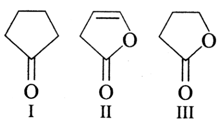
41. If optical rotation produced by  is 36° then that produced by  is :

(A) –36° (B) 0° (C) +26° (D) Unpredictable

Solution :

[B] [OIA]

Hence the answer is (B).

42. 

Among these compounds the order of enol content should be :

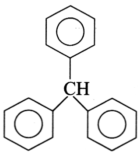
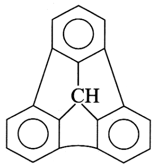
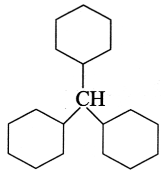
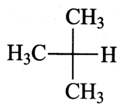
(A) I > II > III (B) III > II > I (C) II > I > III (D) II > III > I

Solution :

(C) II > I > III [Aromatic]

Hence the answer is (C).

43. Identify the compound which contain most acidic hydrogen :

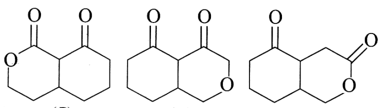
(A)  (B)  (C)  (D) 

Solution :

$\boxed{B}$ → Planar Molecule, easily delocalised

Hence the answer is (B).

44. Compare acidic strength of the following compound.

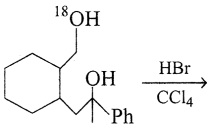


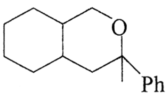
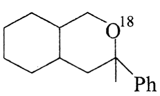
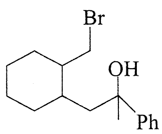
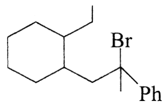
(A) P > Q > R (B) Q > P > R (C) R > P > Q (D) R > Q > P

Solution :

$\boxed{B}$ ∵ carbanion (C.B) stability

Hence the answer is (B).

45.  Major product obtained in this reaction is :

(A)  (B)  (C)  (D) 

Solution :

(B) ∵ [Intramolecular Cyclization]

Hence the answer is (B).

Integer Type :

46. Consider the following reversible system :

$A(g) + 2B(g) \rightleftharpoons AB_2(g)$; $K_C = 1/2$

The above equilibrium is established in a 1.0L flask and at equilibrium 2 moles of each A and B are present. If 2.0 moles of B are added further, how many moles of AB_2 should be added so that moles of A does not change ?

Solution :

Hence the answer is (12).

47. What weight of solute (M.wt. 60) is required to dissolve in 180 g of water to reduce the vapour pressure to 4/5th of pure water ?

Solution :

$$\because \frac{P^\circ - P_s}{P_s} = \frac{w_A}{w_B} \times \frac{m_B}{m_A}$$

$$P_S = \frac{4P^\circ}{5}, m_A = 60, w_A = ?, w_B = 180\text{ g}, m_B = 18$$

$$\therefore \frac{P^\circ - \frac{4P^\circ}{5}}{\frac{4P^\circ}{5}} = \frac{w_A \times 18}{60 \times 180}$$

$$\therefore w = \frac{60 \times 180}{4 \times 18} = 150\text{ g}$$

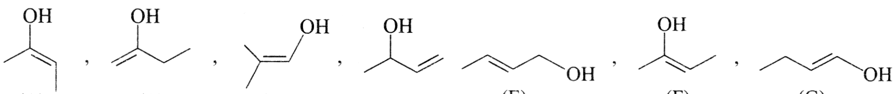
Hence the answer is (150).

48. Certain amount of a non-ideal gas is changed from state (500 K, 5 atm, 2L) to (150 K, 2 atm, 1L). If the change in internal energy is 14 L-atm, change in enthalpy in L-atm unit is

Solution :

Hence the answer is (6).

49. How many compounds A through G are enol tautomers of 2-butanone ?

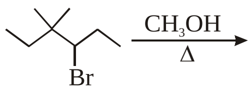


(A) , (B) , (C) , (D) (E) , (F) , (G)

Solution :

Hence the answer is (3).

50. Find out numbers of possible E_1 products from following reaction.



Solution :

(4) [Including C.I.]

$E_1 \rightarrow$ via $\oplus \xrightarrow{R}$ M.S

[All possible Product Counted including stereoisomer]

Hence the answer is (4).

❐❐❐❐❐❐❐

51. Let a, b, c be the real numbers. Then following system of equations in x, y and z,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,\ \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$$ has :

(A) no solution
(B) unique solution
(C) infinitely many solution
(D) finitely many solutions

Solution :

Let $\frac{x^2}{a^2}=x,\ \frac{y^2}{b^2}=y,\ \frac{z^2}{c^2}=z,$

Then the given system of equations is

$$x+y-z=1$$
$$x-y+z=1$$
$$-x+y+z=1$$

$$\text{Determinant}=\begin{vmatrix}1 & 1 & -1\\ 1 & -1 & 1\\ -1 & 1 & 1\end{vmatrix}\neq 0$$

So, unique solution.

Hence the answer is (B).

52. The curve $y=x^3+x^2-x$ has two horizontal tangents. The distance between these two horizontal lines, is $\frac{p}{q}$ (where $p,q\in I$ and H.C.F. of p and q is unity). Find $(p+q)$:

(A) 58
(B) 59
(C) 60
(D) 61

Solution :

$$y=x^3+x^2-x\ ;\ \frac{dy}{dx}=3x^2+2x-1$$

$$\frac{dy}{dx}=(3x-1)(x+1)$$

$$\frac{dy}{dx}=0\ ;\ x=-1,\frac{1}{3}$$

$$y(-1)=-1+1+1=1$$

$$y\left(\frac{1}{3}\right)=\frac{1}{27}+\frac{1}{9}-\frac{1}{3}=\frac{1+3-9}{27}=-\frac{5}{27}$$

So, $y(-1)-y\left(\frac{1}{3}\right)=1+\frac{5}{27}=\frac{32}{27}=\frac{p}{q}$

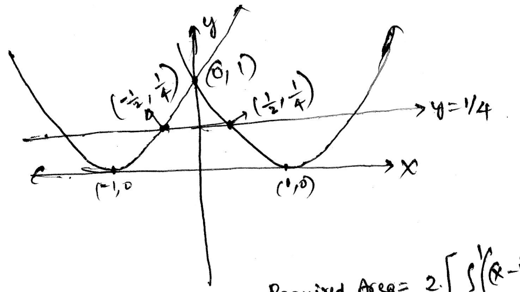
So, $p+q=59$

Hence the answer is (B).

53. The area bounded by the parabolas $y=(x+1)^2$ and $y=(x-1)^2$ and the line y = 1/4 is

(A) 4 sq. units (B) 1/6 sq. units (C) 4/3 sq. units (D) 1/3 sq. units

Solution :



Required Area $= 2\left[\int_0^{1/2}\left((x-1)^2-\frac{1}{4}\right)dx\right]$

$= 2\left[\left(\frac{(x-1)^3}{3}-\frac{x}{4}\right)\right]_0^{1/2}$

$= 2\left[\left(-\frac{1}{24}-\frac{1}{8}\right)-\left(-\frac{1}{3}\right)\right] = \frac{1}{3}$

Hence the answer is (D).

54. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is

(A) $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$ (B) $x^2\sec^2\theta - y^2\operatorname{cosec}^2\theta = 1$

(C) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ (D) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

Solution :

As the hyperbola is confocal with the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$, the equation of the hyperbola will be of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Transverse axis $= 2a = 2\sin\theta \Rightarrow a = \sin\theta$.

Now, $b^2 = a^2(e^2-1) = \sin^2\theta(e^2-1)$ ... (1)

But for the ellipse, foci $= (\pm 2e', 0)$, where $3 = 4(1-e'^2)$

$\therefore$ foci $= (\pm 1, 0)$

For the ellipse, the distance between foci = 2 and for the hyperbola it is $2ae$. So, $2 = 2ae$ $\therefore e = \frac{1}{a} = \frac{1}{\sin\theta}$

$\therefore$ (1) $\Rightarrow b^2 = \sin^2\theta\left(\frac{1}{\sin^2\theta - 1}\right) = 1-\sin^2\theta = \cos^2\theta$.

$\therefore$ the hyperbola has the equation $\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$.

Hence the answer is (C).

55. Let $I=\int\frac{e^x}{e^{4x}+e^{2x}+1}dx,\ J=\int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$

Then, for an arbitarary constant c, the value of J - I equals

(A) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+c$ (B) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+c$

(C) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+c$ (D) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+c$

Solution :

$$J-I=\int\left(\frac{e^{3x}}{1+e^{2x}+e^{4x}}-\frac{e^{x}}{e^{4x}+e^{2x}+1}\right)dx=\int\frac{(e^{2x}-1)\,e^{x}}{e^{4x}+e^{2x}+1}\,dx$$

$$=\int\frac{z^2-1}{z^4+z^2+1}\,dz,\ \text{(putting } e^x=z)$$

$$=\int\frac{1-\frac{1}{z^2}}{z^2+\frac{1}{z^2}+1}\,dz=\int\frac{d\left(z+\frac{1}{z}\right)}{\left(z+\frac{1}{z}\right)^2-1}=\frac{1}{2}\log\frac{\left(z+\frac{1}{z}\right)-1}{\left(z+\frac{1}{z}\right)+1}+c$$

$$=\frac{1}{2}\log\frac{e^x+e^{-x}-1}{e^x+e^{-x}+1}+c=\frac{1}{2}\log\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}+c.$$

Hence the answer is (C).

56. Number of roots of the equation $z^{10}-z^5-992=0$, where real parts are negative, is

(A) 3 (B) 4 (C) 5 (D) 6

Solution :

$z^{10}-z^5-992=0$

$z^{10}-32z^5+31z^5-992=0$ | $992=31\times 32$

$z^5(z^5-32)+31(z^5-32)=0$

$(z^5+31)(z^5-32)=0 \Rightarrow z^5=-31,\ 32$

But the real part is negative, therefore $z^5=32$ doesnot hold.

Hence the answer is (C).

57. The position vectors of the vertices A, B, C of a triangle are $\vec{i}-\vec{j}-3\vec{k}$, $2\vec{i}+\vec{j}-2\vec{k}$ and $-5\vec{i}+2\vec{j}-6\vec{k}$ respectively. The length of the bisector AD of the angle BAC where D is on the line segment BC, is

(A) $\frac{15}{2}$ (B) $\frac{1}{4}$ (C) $\frac{11}{2}$ (D) none of these

Solution :

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(2\vec{i}+\vec{j}-2\vec{k})-(\vec{i}-\vec{j}-3\vec{k})=\vec{i}+2\vec{j}+\vec{k}.$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(-5\vec{i}+2\vec{j}-6\vec{k})-(\vec{i}-\vec{j}-3\vec{k})=-6\vec{i}+3\vec{j}-3\vec{k}.$

A vector along the bisector of the angle BAC

$$=\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}+\frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}=\frac{\vec{i}+2\vec{j}+\vec{k}}{\sqrt{1^2+2^2+1^2}}+\frac{-6\vec{i}+3\vec{j}-3\vec{k}}{\sqrt{(-6)^2+3^2+(-3)^2}}$$

$$=\frac{1}{\sqrt{6}}(\vec{i}+2\vec{j}+\vec{k})+\frac{1}{3\sqrt{6}}(-6\vec{i}+3\vec{j}-3\vec{k})=\frac{1}{3\sqrt{6}}(-3\vec{i}+9\vec{j})=\frac{-\vec{i}+3\vec{j}}{\sqrt{6}}.$$

$\therefore$ the unit vector along AD $=\dfrac{-\vec{i}+3\vec{j}}{\sqrt{10}}.$

$\therefore\ \overrightarrow{AD}=\dfrac{-\vec{i}+3\vec{j}}{10}AD.$

As D is on BC, $\overrightarrow{BD}=t\overrightarrow{BC}.$

$\therefore\ \overrightarrow{BA}+\overrightarrow{AD}=t(\overrightarrow{BA}+\overrightarrow{AC})$

or $\quad -\vec{i}-2\vec{j}-\vec{k}+\frac{-\vec{i}+3\vec{j}}{10}AD=t\{-\vec{i}-2\vec{j}-\vec{k}-6\vec{i}+3\vec{j}-3\vec{k}\}$

$$=t(-7\vec{i}+\vec{j}-4\vec{k})$$

$\Rightarrow \quad -1-\frac{AD}{10}=-7t, \quad -2+\frac{3}{10}AD=t, \quad -1=-4t.$

$\therefore \quad t=\frac{1}{4}$

$\therefore \quad -1-\frac{AD}{10}=-\frac{7}{4} \quad$ or $\quad \frac{AD}{10}=\frac{3}{4}$

$\therefore \quad AD=\frac{15}{2}.$

Hence the answer is (A).

58. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30^0 then $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 3

Solution :

$\vec{a}\times\vec{b}=(2\vec{i}+\vec{j}-2\vec{k})\times(\vec{i}+\vec{j})=2\vec{i}-2\vec{j}+\vec{k}.$

$\therefore \quad |(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}|\,|\vec{b}|\sin 30°$

$$=\sqrt{2^2+(-2)^2+1^2}\cdot|\vec{c}|\cdot\frac{1}{2}=\frac{3}{2}|\vec{c}|.$$

Now, $|\vec{c}-\vec{a}|=2\sqrt{2} \quad \Rightarrow (\vec{c}-\vec{a})^2=8$

or $\quad |\vec{c}|^2+|\vec{a}|^2-2\vec{c}\cdot\vec{a}=8$

or $\quad |\vec{c}|^2+(\sqrt{2^2+1^2+(-2)^2})^2-2|\vec{c}|=8$

or $\quad |\vec{c}|^2-2|\vec{c}|+1=0$

$\therefore \quad |\vec{c}|=1$

$\therefore \quad |(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}|\,|\vec{c}|\sin 30°=\frac{3}{2}.$

Hence the answer is (B).

59. $\lim\limits_{n\to\infty}\left\{\frac{n!}{(kn)^n}\right\}^{1/n}$, where $k\neq 0$ is a constant and $n\in N$, is equal to

(A) ke (B) $k^{-1}.e$ (C) ke^{-1} (D) $k^{-1}.e^{-1}$

Solution :

$$\text{Limit}=\lim_{n\to\infty}\frac{1}{k}\left\{\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdot\ldots\cdot\frac{n}{n}\right\}^{1/n}$$

$$=\frac{1}{k}e^{\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=1}^{n}\log\frac{r}{n}}=\frac{1}{k}e^{\int_0^1\log x\,dx}=\frac{1}{k}e^{[x\log x]_0^1-\int_0^1 x\cdot\frac{1}{x}dx}$$

$$=\frac{1}{k}e^{-1} \quad \left(\because \lim_{x\to 0}x\log x=\lim_{x\to 0}\frac{\log x}{1/x}=\lim_{x\to 0}\frac{1/x}{-1/x^2}=\lim_{x\to 0}(-x)=0\right).$$

Hence the answer is (D).

60. The value of $\int_1^2 \left[f\{g(x)\}\right]^{-1}.f'\{g(x)\}.g'(x)dx$, where g(1) = g(2), is equal to

(A) 1 (B) 2 (C) 0 (D) none of these

Solution :

Let $g(x) = z$. Then $I = \int_{g(1)}^{g(2)} \frac{1}{f(z)} \cdot f'(z)\,dz = \left[\log f(z)\right]_{g(1)}^{g(2)}$

$= \log f\{g(2)\} - \log f\{g(1)\} = 0 \quad [\because g(1) = g(2)].$

Hence the answer is (C).

61. 6 ordinary dice are rolled. The probability that at least half of them will show at least 3 is

(A) $41\times\frac{2^4}{3^6}$ (B) $\frac{2^4}{3^6}$ (C) $20\times\frac{2^4}{3^6}$ (D) none of these

Solution :

The probability of getting at least 3 in a throw $= \frac{4}{6} = \frac{2}{3}$.

$\therefore$ the required probability

$= {}^6C_3 \cdot \left(\frac{2}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^3 + {}^6C_4 \cdot \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \cdot \left(\frac{2}{3}\right)^5 \cdot \frac{1}{3} + {}^6C_6 \cdot \left(\frac{2}{3}\right)^6.$

Hence the answer is (A).

62. The sum of infinite terms of a decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval [-2, 3] and the difference between the first two terms is $f'(0)$. Then the common ratio of the GP is

(A) $-\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{4}{3}$

Solution :

Let the GP be $a, ar, ar^2, \ldots$ $(0 < r < 1)$. From the question,

$\frac{a}{1-r} = 3^3 + 3.3 - 9$

$\{\because f'(x) = 3x^2 + 3 > 0$; so, $f(x)$ is monotonically increasing;

$\therefore f(3)$ is the greatest value in $[-2, 3]$.$\}$

Also, $f'(0) = 3$. So, $a - ar = 3$.

Solving, $a = 27(1-r)$ and $a(1-r) = 3$ we get $r = \frac{2}{3}, \frac{4}{3}$. But $r < 1$.

Hence the answer is (C).

63. The sum $1.{}^{20}C_1 - 2.{}^{20}C_2 + 3.{}^{20}C_3 - .... - 20.{}^{20}C_{20}$ is equal to

(A) 2^{19} (B) 0 (C) $2^{10} - 1$ (D) none of these

Solution :

Using $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$,

sum $= 20\{{}^{19}C_0 - {}^{19}C_1 + {}^{19}C_2 - \ldots - {}^{19}C_{19}\} = 20 \times 0 = 0.$

Hence the answer is (B).

64. Which of the following statement are not logically equivalent

(A) $\sim(p\vee\sim q)$ and $(\sim p \vee q)$ (B) $\sim(p\to q)$ and $(p\wedge\sim q)$

(C) $(p\to q)$ and $(\sim q \to \sim p)$ (D) $(p\to q)$ and $(\sim p \wedge q)$

Solution :

Draw truth table

Hence the answer is (D).

65. The median and standard deviation (S.D.) of a distribution will be, If each term is increaed by 2 -

(A) median and S.D. will increased by 2

(B) median will increased by 2 but S.D. will remain same

(C) median will remain same but S.D. will increased by 2

(D) median and S.D. will remain same

Solution :

Medium, Mean, Mode increases by 2 but standard deviation and variance remans same.

Hence the answer is (B).

66. Let us consider a function $f(x) = x^2 + \ln\left(\frac{\pi + x}{\pi - x}\right)\cos x + \ln\left(\frac{\pi - x}{\pi + x}\right)$

if f (10) = 100, then find f (-10).

(A) 99 (B) 100 (C) 101 (D) 10000

Solution :

$f(x) + f(-x) = 2x^2$

$f(-10) = 200 - 100 = 100$

Hence the answer is (B).

67. The sum of squares of the roots satisfying the equation $\log_{\pi^2}\left(\sin^{-1} x\right) + \log_{\pi^2}\left(\cos^{-1} x\right) = 1 - \log_{\pi^2} 18$

(A) 1 (B) 2 (C) 4 (D) none of these

Solution :

$$\log_{\pi^2}(\sin^{-1} x)(\cos^{-1} x) = \log_{\pi^2}\frac{\pi^2}{18}$$

$$\sin^{-1} x \cos^{-1} x = \frac{\pi^2}{18}$$

$$\Rightarrow \quad \sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right) = \frac{\pi^2}{18}$$

$$(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{18} = 0$$

$$\left(\sin^{-1} x - \frac{\pi}{3}\right)\left(\sin^{-1} x - \frac{\pi}{6}\right) = 0$$

$$x_1 = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$x_2 = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$x_1^2 + x_2^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{4}{4} = 1.$$

Hence the answer is (A).

68. Number of points where the function $f(x) = (x^2 - 1)\left|(x^2 - x - 2)\right| + \sin(|x|)$ is not differentiable is :

(A) 0 (B) 1 (C) 2 (D) 3

Solution :

$f(x) = (x^2 - 1)|x^2 - x - 2| + \sin(|x|)$

$= (x-1)(x+1)|(x-2)(x+1)| + \sin(|x|)$

Not derivable at $x = 0$ and 2.

Hence the answer is (C).

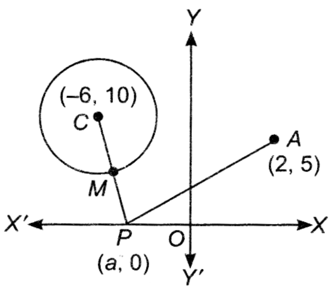
69. The length of the shortest path that begins at the point (2,5) touches the x-axis and then ends at a point on the circle $x^2 + y^2 + 12x - 20y + 120 = 0$, is :

(A) 13 (B) $4\sqrt{10}$ (C) 15 (D) $6 + \sqrt{89}$

Solution :

$x^2 + y^2 + 12x - 20y + 120 = 0$

So, centre = (−6, 10), radius = $\sqrt{(36 + 100 - 120)} = 4$



$L = AP + PMC$

$L = AP + PC - CM$

$L = \sqrt{[(a-2)^2 + 5^2]} + \sqrt{[(a+6)^2 + 10^2]} - 4$

$$\frac{dL}{da} = \frac{2(a-2)}{2\sqrt{[(a-2)^2 + 5^2]}} + \frac{2(a+6)}{2\sqrt{[(a+6)^2 + 10^2]}}$$

$\frac{dL}{da} = 0$; $a = -\frac{2}{3}$ or 10 (not possible),

$$L_{\min} = \sqrt{\left(\frac{64}{9} + 25\right)} + \sqrt{\left(\frac{256}{9} + 100\right)} - 4$$

$$= \frac{17}{3} + \frac{\sqrt{(1156)}}{3} - 4$$

$$= \frac{17}{3} + \frac{34}{3} - 4$$

$= 17 - 4 = 13.$

Hence the answer is (A).

70. The arithmetic mean of the ordinates of the feet of the normals from (3,5) to the parabola $y^2 = 8x$ is

(A) 4 (B) 0 (C) 8 (D) none of these

Solution :

The sum of he ordinates of the three normals $= y_1 + y_2 + y_3 = 0$

Hence the answer is (B).

Integer Type :

71. If $\sum_{k=1}^{\infty}\frac{k^2}{5^k}=\frac{a}{b}$, where a, b are relatively prime positive integers, than the value of b-2a is equal to :

Solution :

$$\sum_{k=1}^{\infty}\frac{k^2}{5^k}=\frac{a}{b}$$

$$\text{let } S=\frac{1}{5}+\frac{4}{5^2}+\frac{9}{5^3}+\frac{16}{5^4}+\cdots\infty$$

$$\frac{1}{5}S=\frac{1}{5^2}+\frac{4}{5^3}+\frac{9}{5^4}+\cdots\infty$$

$$\ominus$$

$$\frac{4}{5}S=\frac{1}{5}+\frac{3}{5^2}+\frac{5}{5^3}+\frac{7}{5^4}+\cdots\infty$$

$$\frac{1}{5}\cdot\frac{4}{5}S=\frac{1}{5^2}+\frac{3}{5^3}+\frac{5}{5^4}+\cdots\infty$$

$$\ominus$$

$$\frac{4}{5}S\left(1-\frac{1}{5}\right)=\frac{1}{5}+\frac{2}{5^2}+\frac{2}{5^3}+\frac{2}{5^4}+\cdots\infty$$

$$\frac{4}{5}\cdot\frac{4}{5}S=\frac{1}{5}+2\frac{\left(\frac{1}{5^2}\right)}{1-\frac{1}{5}}=\frac{1}{5}+\frac{1}{10}$$

$$\frac{16}{25}S=\frac{3}{10}\Rightarrow S=\frac{15}{32}=\frac{a}{b}$$

$$b-2a=32-2\cdot15=30.$$

Hence the answer is (2).

72. For all real values of a and b, lines

$(2a+b)x+(a+3b)y+(b-3a)=0$ and $mx+2y+6=0$ are concurrent. Then |m| is equal to

Solution :

$(2a+b)x+(a+3b)y+(b-3a)=0$

$\Rightarrow a(2x+y-3)+b(x+3y+1)=0$ represents a family of lines passes through point of intersection of

$2x+y-3=0$ & $x+3y+1=0$ i.e, $(2,-1)$

$\Rightarrow mx+2y+6=0$ passes through $(2,-1)$

$2m-2+6=0 \Rightarrow m=-2$

$\Rightarrow |m|=2$

Hence the answer is (2).

73. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2,5 and 7. The smallest value of n for which this is possible is

Solution :

Distinct n-digit numbers which can be formed using digits 2, 5 and 8 are 3^n. We have to find 'n' so that

$3^n \geq 900$

$\Rightarrow 3^{n-2} \geq 100$

$n-2 \geq 5 \Rightarrow n \geq 7$

So the least value of 'n' is 7.

Hence the answer is (7).

74. If x > 0 and x, [x], {x} when [.] denotes greatest integer function and {.} denotes fractional part function, are in A.P. then the number of possible values of x is.

Solution :

$2\{x\} = x + [x]$

$2\{x\} = [x] + \{x\} + [x]$

$\Rightarrow \{x\} = 2[x]$

$0 \leq \{x\} < 1$

$0 \leq 2[x] < 1$

$0 \leq [x] < 1/2 \quad \Rightarrow [x] = 0$

$\{x\} = 0$ ✓

$x = [x] + \{x\} = \underline{\underline{0}}$.

Hence the answer is (1).

75. Given two vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, $b = -2\hat{i} + 2\hat{j} - \hat{k}$ and $\lambda = \left|\dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projection of } \vec{b} \text{ on } \vec{a}}\right|$ then 3λ is equal to.

Solution :

$\lambda = \left| \dfrac{\text{the projection of } \vec{a} \text{ on } \vec{b}}{\text{the projection of } \vec{b} \text{ on } \vec{a}} \right| = \left| \dfrac{\vec{a} \cdot \hat{b}}{\vec{b} \cdot \hat{a}} \right| = \dfrac{|\vec{a}|}{|\vec{b}|}$

$\lambda = \dfrac{\sqrt{4+9+36}}{3} \Rightarrow \boxed{3\lambda = 7}$

Hence the answer is (7).

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