SOLUTIONS

WITH

ANSWER KEY

ANTS-FT # 21

DROPPER ENGINEERING

(PHYSCIS, CHEMISTRY & MATHS)

TARGET : JEE (MAIN + ADVANCED) - 2020

Exam. Date : 21-06-2020

POTENTIAL & CONCEPT EDUCATIONS
NEET | AIIMS JEE | KVPY | OLYMPIAD
JOINING HANDS FOR A BETTER TOMORROW

ANSWER KEYS FOR ANTS-FT # 21 (TARGET – JEE-MAIN-2020)

DATE : 21-06-2020

ANSWERS [PHYSICS]

1. A	2. A	3. A	4. B	5. D	6. D	7. D	8. B	9. B	10. A
11. A	12. B	13. C	14. A	15. D	16. A	17. C	18. D	19. B	20. C
21. (12)	22. (84)	23. (4)	24. (7)	25. (20)					

ANSWERS [CHEMISTRY]

26. B	27. A	28. D	29. A	30. C	31. A	32. B	33. D	34. D	35. B
36. C	37. A	38. B	39. C	40. B	41. D	42. A	43. C	44. B	45. A
46. (12.0)		47. (80)		48. (23.7)		49. (1)	50. (0.51)		

ANSWERS [MATHS]

51. D	52. B	53. C	54. B	55. A	56. A	57. D	58. B	59. C	60. C
61. C	62. D	63. B	64. B	65. C	66. D	67. C	68. C	69. C	70. D
71. (1000)		72. (0.167)		73. (151)		74. (0.75)		75. (4)	

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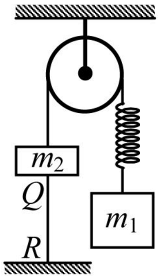
1. In the shown system, $m_1 > m_2$. Thread QR is holding the system. If this thread is cut, then just after cutting.



(A) acceleration of mass m_1 is zero and that of m_2 is directed upward

(B) acceleration of mass m_2 is zero and that of m_1 is directed downward

(C) acceleration of both the blocks will be same

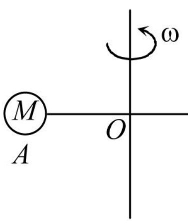
(D) acceleration of system is given by $k\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$, constant.

Solution :

On cutting the string QR, the resultant force on m_1 remains zero because its weight m_1g is balanced by the tension in the spring but on block m_2 a resultant upward force $(m_1 - m_2)g$ is developed. Thus block m_1 will have no resultant acceleration whereas m_2 does have an upward acceleration given by $\frac{(m_1 - m_2)g}{m_2}$.

Hence the answer is (A).

2. Two balls of mass M = 9 g and m = 3 g are attached by massless threads AO and OB. The length AB is 1 m. They are set in rotational motion in a horizontal plane about a vertical axis at O with constant angular velocity ω. The ratio of length AO and OB $\left(\frac{AO}{OB}\right)$ for which the tension in threads are same will be



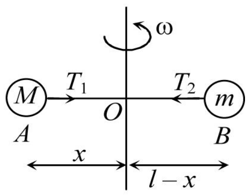
(A) $\frac{1}{3}$ (B) 3 (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

Solution :

$T_1 = T_2$

$\Rightarrow M\omega^2 x = m\omega^2(l - x)$

$x = \frac{ml}{M + m}$



$\frac{AO}{OB} = \frac{x}{l - x} = \frac{m}{M} = \frac{3}{9} = \frac{1}{3}$

Hence the answer is (A).

3. A constant power is supplied to a rotating disc. Angular velocity (ω) of disc varies with number of rotations (n) made by the disc as

(A) $\omega \propto n^{1/3}$ (B) $\omega \propto n^{3/2}$ (C) $\omega \propto n^{2/3}$ (D) $\omega \propto n^2$

Solution :

Since, $P = \tau\omega =$ constant $\Rightarrow$ $\alpha\omega = c$ (constant)

$\Rightarrow \omega^2 \frac{d\omega}{d\theta} = c \Rightarrow \omega \propto \theta^{1/3}$

$\therefore \omega \propto n^{1/3}$ (as $\theta \propto n$)

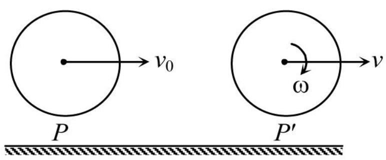
Hence the answer is (A).

4. A ring of mass m is placed on a very rough horizontal surface with its plane vertical. A horizontal impulse J is applied on the ring of mass m along a line passing through its centre. The linear velocity of the centre of the ring once it starts pure rolling is

(A) $\frac{J}{m}$ (B) $\frac{J}{2m}$ (C) $\frac{2J}{m}$ (D) $\frac{J}{3m}$

Solution :

Let v be the velocity of centre of mass of ring just after the impulse is applied and v is its velocity in pure rolling.



$v_0 = \frac{J}{m}$

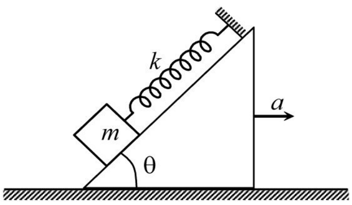
Conserving angular momentum about point of contact with ground

$mv_0r = mvr + I_{cm}\omega$

$\Rightarrow v = \frac{v_0}{2} = \frac{J}{2m}$

Hence the answer is (B).

5. A spring-block system is kept on a smooth wedge of inclination θ as shown in figure. The mass of the block is m, spring constant of the spring is k. The wedge is moving with constant acceleration a. The time period for small oscillation of block is (assuming at all times mass m remains in contact with the wedge)



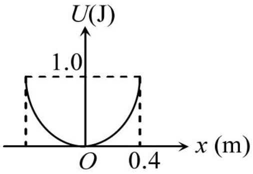
(A) $2\pi\sqrt{\frac{m}{k\sin\theta}}$ (B) $2\pi\sqrt{\frac{m}{k\sin^2\theta}}$ (C) $2\pi\sqrt{\frac{m\sin\theta}{k}}$ (D) $2\pi\sqrt{\frac{m}{k}}$

Solution :

Time period of spring block system does not depend on the effective g

Hence the answer is (D).

6. A particle of mass 2 kg moves in simple harmonic motion and its potential energy U varies with position x as shown. The period of oscillation of the particle is (in second)



(A) $\frac{2\pi}{5}$ (B) $\frac{2\sqrt{2}\,\pi}{5}$ (C) $\frac{\sqrt{2}\,\pi}{5}$ (D) $\frac{4\pi}{5}$

Solution :

$\omega = \frac{5}{2}$ radian/sec, $T = \frac{2\pi}{\omega} = \frac{4\pi}{5}$

Hence the answer is (D).

7. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

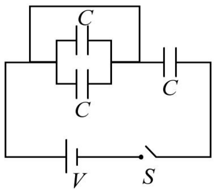
(A) $\frac{2a\lambda}{\varepsilon_0}$ (B) $\frac{\sqrt{2}\lambda a}{\varepsilon_0}$ (C) $\frac{6\lambda a}{\varepsilon_0}$ (D) $\frac{\sqrt{3}\lambda a}{\varepsilon_0}$

Solution :

The maximum length of the string which can fit into the cube is $\sqrt{3}a$, equal to its body diagonal. The maximum charge inside the cube is , and hence the maximum flux through the cube is $\frac{\sqrt{3}\lambda a}{\varepsilon_0}$

Hence the answer is (D).

8. In the given circuit, find the heat generated if switch S is closed.



(A) $\frac{3}{2}CV^2$ (B) 2 CV^2 (C) CV^2 (D) $\frac{1}{3}CV^2$

Solution :

$C_{eq} = C$

So, work done by battery = CV^2

Heat generated = $\frac{1}{2}CV^2$

Hence the answer is (B).

9. If the flux of magnetic induction through a coil of resistance R and having n turns changes from ϕ_1 to ϕ_2, then the magnitude of the charge that passes through the coil is

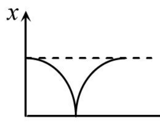
(A) $\frac{(\varphi_2 - \varphi_1)}{R}$ (B) $\frac{n(\varphi_2 - \varphi_1)}{R}$ (C) $\frac{(\varphi_2 - \varphi_1)}{nR}$ (D) $\frac{nR}{(\varphi_2 - \varphi_1)}$

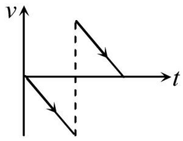
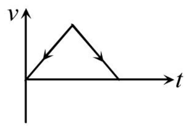
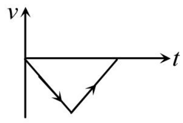
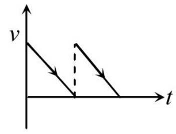
Solution :

Induced is emf is $|e| = n\frac{\Delta\varphi}{\Delta t}$.

Hence the answer is (B).

10. Position-time curve of a body moving along a straight line is shown in figure. The velocity-time curve for the motion of the particle will be



(A)  (B)  (C)  (D) 

Solution :

This is the situation similar to elastic collision of ball impinging on floor and bouncing back.

Hence the answer is (A).

11. A 4.5 cm-metallic sphere (work function = 1.1 eV) is exposed to intense light of wavelength 400 nm. After some time, it is observed that photoemission stops. The final charge on the sphere is (assume that the value of hc = 12400 eV-Å)

(A) 10^{-11}C (B) 1.55×10^{-11}C (C) 0.55×10^{-11}C (D) 2.1×10^{-11}C

Solution :

The capacitance of the sphere, $C = 4\pi\varepsilon_0 r$

$$= \frac{1}{9}\times 10^{-9}\times\frac{4.5}{100} = 0.5\times 10^{-11}\text{F}$$

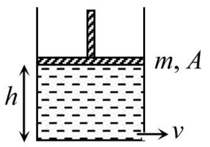
After exposure to the light photo emission stops when the final potential of the sphere equals the stopping potential:

$$V_{stop} = \frac{12400}{4000} - 1.1 = 2\,\text{V}$$

The charge on the sphere = $CV_{stop} = 10^{-11}$C

Hence the answer is (A).

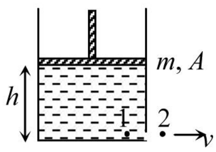
12. A cylindrical vessel contains a liquid of density ρ upto a height h. The liquid is closed by a piston of mass m and area of cross-section A. There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is: (neglect presence of atmosphere)



(A) $\sqrt{2gh}$ (B) $\sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$ (C) $\sqrt{2\left(gh + \frac{mg}{A}\right)}$ (D) $\sqrt{2gh + \frac{mg}{A}}$

Solution :

Applying Bernoulli's theorem at 1 and 2



$$\rho gh + \frac{mg}{A} = \frac{1}{2}\rho v^2$$

or

$$v = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$$

Hence the answer is (B).

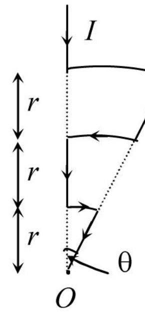
13. The radii of two soap bubbles are R_1 and R_2 respectively. The ratio of masses of air in them will be

(A) $\frac{R_1^3}{R_2^3}$ (B) $\frac{R_2^3}{R_1^3}$ (C) $\left(\frac{P+\frac{4T}{R_1}}{P+\frac{4T}{R_2}}\right)\frac{R_1^3}{R_2^3}$ (D) $\left(\frac{P+\frac{4T}{R_2}}{P+\frac{4T}{R_1}}\right)\frac{R_2^3}{R_1^3}$

Solution :

Hence the answer is (C).

14. Shown in the figure is a conductor carrying a current I. The magnetic field intensity at the point O (common centre of all the three arcs) is (θ in radian)



(A) $\frac{5\mu_0 I\theta}{24\pi r}$ (B) $\frac{\mu_0 I\theta}{24\pi r}$ (C) $\frac{11\mu_0 I\theta}{24\pi r}$ (D) zero

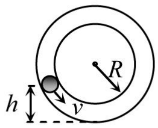
Solution :

Since magnetic field at the centre of an arc is equal to

Hence, net $B = \quad = \frac{5\mu_0 I\theta}{24\pi r}$

Hence the answer is (A).

15. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube? Radius of the tube is R. (Assume radius of cross-section of tube is negligible in comparison to R)



(A) $\sqrt{g(2R-h)}$ (B) $\frac{5}{2}R$ (C) $\sqrt{g(5R-2h)}$ (D) $\sqrt{2g(2R-h)}$

Solution :

For minimum *v*, velocity of ball at the topmost point will be zero.
By conservation of energy,

$$v = \sqrt{2g(2R-h)}$$

Hence the answer is (D).

16. If a man at the equator would weight (3/5)th of his weight, then the angular speed of the earth would be

(A) $\sqrt{\frac{2}{5}\frac{g}{R}}$ (B) $\sqrt{\frac{g}{R}}$ (C) $\sqrt{\frac{R}{g}}$ (D) $\sqrt{\frac{2}{5}\frac{R}{g}}$

Solution :

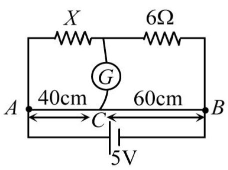
At equator, $g' = g - \omega^2 R$

$\Rightarrow \frac{3}{5}g = g - \omega^2 R$

$\Rightarrow \omega = \sqrt{\frac{2g}{5R}}$

Hence the answer is (A).

17. In the circuit shown, a meter bridge is in its balanced state. The meter bridge wire has a resistance 0.1 ohm/cm. The value of unknown resistance X and the current drawn from the battery of negligible resistance is



(A) 6 Ω, 5 amp (B) 4 Ω, 0.1 amp (C) 4 Ω, 1.0 amp (D) 12 Ω, 0.5 amp

Solution :

Hence the answer is (C).

18. An alternating voltage is given by voltage is given by $e = e_1 \sin\omega t + e_2 \cos\omega t$. Then, the root mean square value of

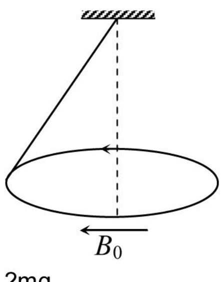
(A) $\sqrt{e_1^2 + e_2^2}$ (B) $e_1 + e_2$ (C) $\sqrt{\frac{e_1 e_2}{2}}$ (D) $\sqrt{\frac{e_1^2 + e_2^2}{2}}$

Solution :

$e = e_0 \sin(\omega t + \varphi)$, $e_{rms} = \frac{e_0}{\sqrt{2}} = \sqrt{\frac{e_1^2 + e_2^2}{2}}$

Hence the answer is (D).

19. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown. A uniform magnetic field B_0 exist in the region to keep the ring in horizontal position, then the current in the ring is (ℓ : length of string)



(A) $\frac{2mg}{RB_0}$ (B) $\frac{mg}{RB_0\pi}$ (C) $\frac{mg}{3\pi RB_0}$ (D) $\frac{mg\ell}{\pi R^2 B_0}$

Solution :

Hence the answer is (B).

20. If a particle is fired vertically upwards from the surface of earth and reaches a height of 6400 km, the initial velocity of the particle is (Assume R = 6400 km and g = 10 ms^{-2})

(A) 4 km/sec (B) 2 km/sec (C) 8 km/sec (D) 16 km/sec

Solution :

According to law of conservation of energy, $\frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$

$$v^2 = \frac{2gh}{1+\frac{h}{R}} = \frac{2\times10\times6.4\times10^6}{1+\frac{R}{R}} = \frac{2\times10\times6.4\times10^6}{2} \quad \therefore v = \sqrt{64\times10^6} = 8\text{ km / sec}$$

Hence the answer is (C).

Integer Type

21. A freshly prepared radioactive source half-life 2hr emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

Solution :

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \text{ or } \frac{1}{64} = \left(\frac{1}{2}\right)^{t/T} \text{ or } \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/T} \text{ or } \frac{t}{T} = 6$$

$\therefore$ $t = 6T$ or $t = 6 \times 2 = 12$

Hence the answer is (12).

22. In Young's double slit experiment when sodium light of wavelength 5893 Å is used, then 62 fringes are seen in the field of view. Instead of sodium light, if violet light of wavelength 4358 Å is used then the number of fringes that will be seen in the field of view will be

Solution :

$$n_1\frac{D\lambda_1}{d} = n_2\frac{D\lambda_2}{d} \Rightarrow n_1\lambda_1 = n_2\lambda_2 \Rightarrow n_2 = \frac{n_1\lambda_1}{\lambda_2} \approx 84$$

Hence the answer is (84).

23. A wire is of mass (0.3 ± 0.003)gm. The radius is (0.5 ± 0.005)mm and length is (6.0 ± 0.06)cm then % error in density is

Solution :

$$\%\text{ error in density} = \left[2\left(\frac{0.005}{0.5}\right) + \frac{0.003}{0.3} + \frac{0.06}{6}\right]\times100 = 4$$

Hence the answer is (4).

24. Two trains, one coming towards and another going away from an observer both at 4 m/s produce a whistle simultaneously of frequency 300 Hz. The number of beats heard by observer will be (velocity of sound = 340 m/s)

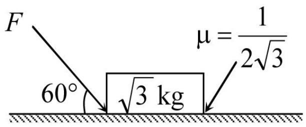
Solution :

$$\text{Number of beats} = f\left(\frac{v}{v-v_s}\right) - f\left(\frac{v}{v+v_s}\right) \approx \frac{2f\,v_s}{v^2}$$

$$= \frac{2\times300\times4}{340} \approx 7$$

Hence the answer is (7).

25. What is the maximum value of the force F such that the block shown in the arrangement, does not move

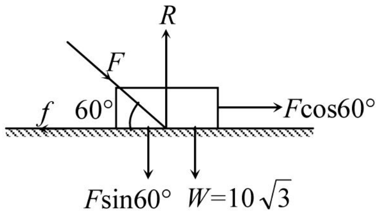


Solution :

$f = \mu R = \mu\left(W + F\sin 60°\right)$

$F\cos 60° = \mu(W + F\sin 60°)$

Substituting $\mu = \frac{1}{2\sqrt{3}}$ and $W = 10\sqrt{3}$, we get F = 20 N



Hence the answer is (20).

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26. $R-\overset{O}{\overset{||}{C}}-Cl + H_2 \xrightarrow[S]{Pd/BaSO_4} RCHO$

the name of the reaction is ?

(A) Cannizzaro's reaction (B) Roseumund reduction

(C) Clemmensen reduction (D) Aldol condensation

Solution :

Theory: name reaction of acid derivate

Hence the answer is (B).

27. $CH_3-CH_2-C\equiv N \xrightarrow[\text{(ii) } H_2O/H^+]{\text{(i) DIBAL–H, } -78^\circ C} A$

Product A is

(A) CH_3CH_2CHO (B) $CH_3CH_2C{=}N{-}OH$ (C) CH_3CH_2COOH (D) $CH_3CH_2CH{=}NH$

Solution :

$$CH_3CH_2C\equiv N \xrightarrow[-78^\circ C]{\text{DIBAL–H}} CH_3CH_2CH=NH \xrightarrow{H_2O/H^+} CH_3CH_2CHO$$

Hence the answer is (A).

28. $\underset{\text{(alkene)}}{A} + m-CPBA \xrightarrow{CH_2Cl_2} B + C$

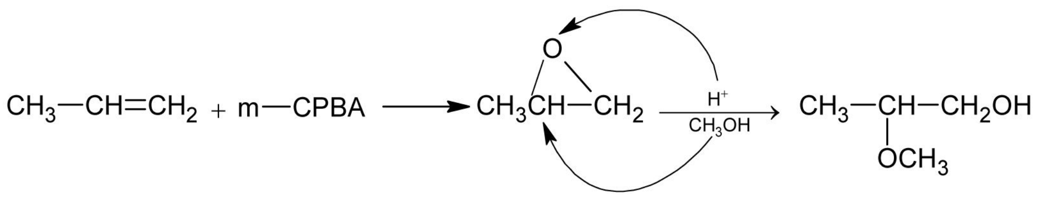
$B + CH_3OH \xrightarrow{H^+} \underset{\text{(major)}}{D} + \underset{\text{(minor)}}{CH_3O-CH_2-\overset{OH}{\overset{|}{C}}HCH_3}$

What is A and D ?

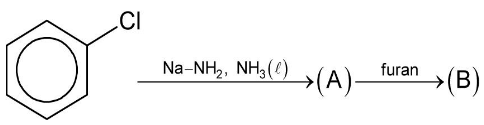
(A) $CH_3CH=CH_2$, $CH_3-\overset{OH}{\overset{|}{C}H}-\overset{OH}{\overset{|}{C}H}-CH_3$ (B) $CH_3CH=CHCH_3$, $CH_3-\overset{OCH_3}{\overset{|}{C}H}-CH_2-OH$

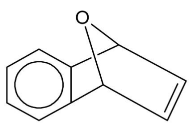
(C) $CH_3CH=CH_2$, $CH_3-\underset{OCH_3}{\underset{|}{\overset{OH}{\overset{|}{C}}}}-CH_3$ (D) $CH_3CH=CH_2$, $CH_3-\overset{OCH_3}{\overset{|}{C}H}\text{-}CH_2-OH$

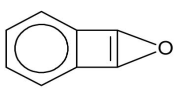
Solution :

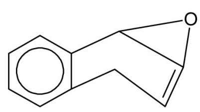


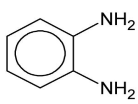
Hence the answer is (D).

29. 

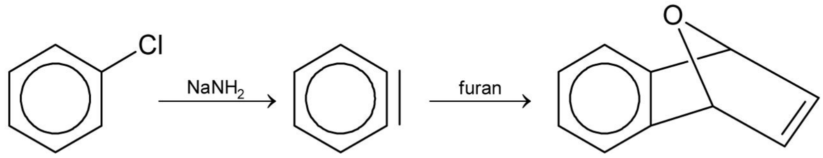
(A) 

(B) 

(C) 

(D) 

Solution :



Hence the answer is (A).

30. Calculate $\Delta G°$ for the following reaction at 298 K :

$CO(g)+\frac{1}{2}O_2(g)\longrightarrow CO_2(g);\ \Delta H° = -282.84\ KJ$

Given: $S^0_{CO_2} = 213.8\ JK^{-1}mol^{-1}$

$S^0_{CO(g)} = 197.9\ JK^{-1}mol^{-1}$

$S^0_{O_2} = 205.0\ JK^{-1}mol^{-1}$

(A) – 86.6 JK^{-1} (B) + 86.6 JK^{-1} (C) – 257.0 KJ (D) + 257.0 KJ

Solution :

$\Delta S° = \Sigma S^0_P - \Sigma S^0_R$

$= 213.8-\left(197.9+\frac{1}{2}\times 205\right) = -86.6\ JK^{-1}$

Now, $\Delta G° = \Delta H° - T\Delta S°$

$= -282.84 - 298 \times (-86.6 \times 10^{-3})$

$= -257.0$ KJ

Hence the answer is (C).

31. What would be the percent hydrolysis of 0.10 (M) N_2H_5Cl, a salt containing the acid ion conjugate to hydrazine base ?

Given: Kb for $N_2H_4 = 9.6 \times 10^{-7}$

(A) 0.0323% (B) 0.00323% (C) 0.323% (D) 3.23%

Solution :

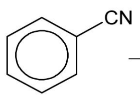
$N_2H_5^+ + H_2O \rightleftharpoons N_2H_4 + H_3O^+$

$K_h = \frac{K_w}{K_b} = \frac{1\times 10^{-14}}{9.6\times 10^{-7}} = 1.04\times 10^{-8}$

$h = \sqrt{\frac{K_h}{c}} = \sqrt{\frac{1.04\times 10^{-8}}{0.10}} = 3.225\times 10^{-4} = 3.23\times 10^{-4}$

$\therefore$ % hydrolysis = 0.0323%

Hence the answer is (A).

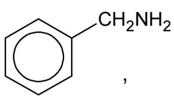
32. 

$\xrightarrow[(2)\ H_3O^+]{(1)\ (i\text{–}Bu)_2AlH} (A)$

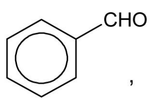
$A + BrCH_2COOC_2H_5 \xrightarrow[Et_2O,\ Heat]{Zn} (B) \xrightarrow{NH_4Cl}$

$(C) + \text{Enantiomer} \xrightarrow[(2)\ Heat]{(1)\ H_3O^+} (D)$

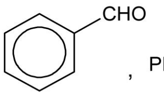
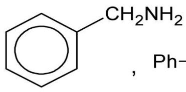
What is (A) and (D)?

(A) 

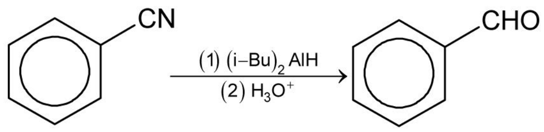
, Ph—CH=CH—NH_2

(B) 

, Ph—CH=CHCOOH

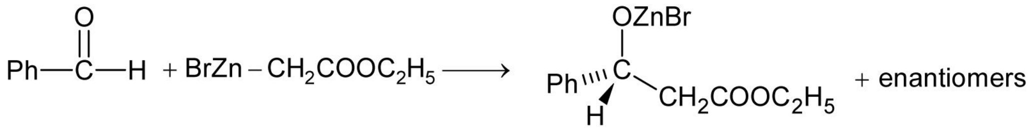
(C)  , $Ph-CH=CHCOOCH_2Br$ (D)  , $Ph-CH=CHCOOC_2H_5$

Solution :

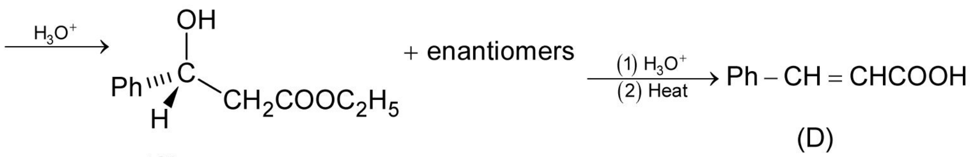


(A)

$$BrCH_2COOC_2H_5 + Zn \xrightarrow[\text{Heat}]{Et_2O} BrZn-CH_2COOC_2H_5$$



(B)



(C) (D)

Hence the answer is (B).

33. $A + NaI \xrightarrow[\text{Heat}]{\text{Acetone}} B + IBr + NaBr$

$$\text{Carboxylic acid} \xleftarrow{KMnO_4,\Delta} B \xrightarrow[H^+/H_2O]{m-CPBA} \text{Enantiomer pair}$$

Identify A and B

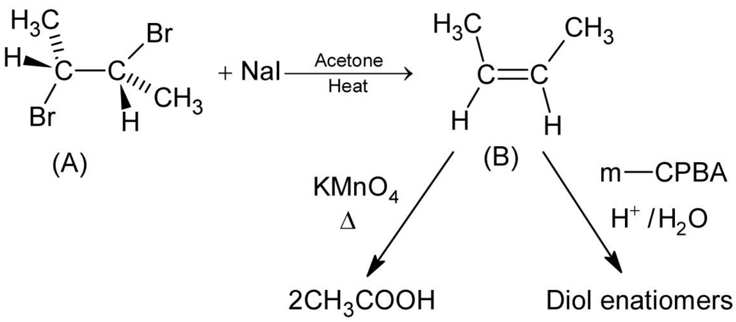
(A) Meso - 2, 3 - Dibromobutane and cis-But - 2 - ene

(B) Meso - 2, 3 - Dibromobutane and trans - But - 2 - ene

(C) (2R, 3R) - 2, 3 - Dibromobutane and But - 1 - ene

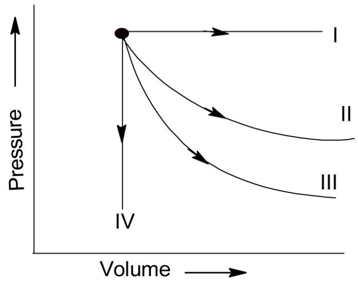
(D) (2R, 3R) - 2, 3 - Dibromobutane and cis - But - 2 - ene

Solution :



Hence the answer is (D).

34. From the given graph below which is correct ?



(A) In process IV, $\Delta V = 0$, compression
(B) In process III, $\Delta E = 0$, expansion
(C) In process I, $\Delta H = 0$, compression
(D) In process II, $\Delta E = 0$, expansion

Solution :

Process – I, Isobaric process
Process – II, Isothermal process
Process – III, Adiabatic process
Process – IV, Isochoric process
All are expansion process

Hence the answer is (D).

35. What is hybridisation of C_2 in following ?

$CH_3–CH=CH–CH_2–CH_3$

1 2 3 4 5

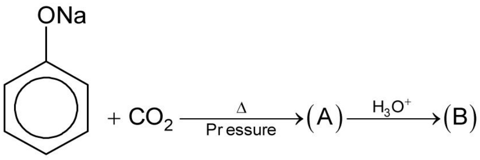
(A) sp^3
(B) sp^2
(C) dsp^2
(D) sp

Solution :

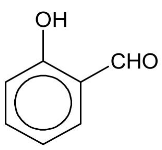
Hybridisation = sp^2

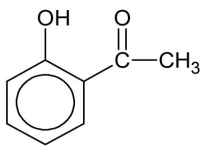
Hence the answer is (B).

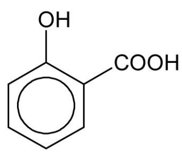
36.

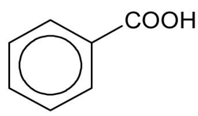


What is B ?

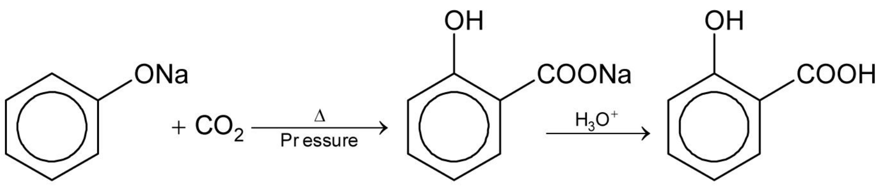
(A) 

(B) 

(C) 

(D) 

Solution :



Hence the answer is (C).

37. $(A) \xrightarrow[HCOOH]{HCHO} (B) \xrightarrow[CH_3OH]{H_2O_2} (C) \xrightarrow{Heat} \underset{(major)}{(D)} + \underset{(minor)}{(E)}$

Compound (A) is primary amine and (B) is tertiary amine. What is major product ?

(A) $CH_3CH_2CH{=}CH_2$ (B) $CH_3CH{=}CHCH_3$

(C) $(CH_3)_2C{=}CH_2$ (D) $(CH_3)_2C{=}CHCH_3$

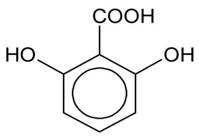
Solution :

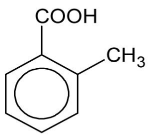
$$\underset{1^\circ\ \text{amine}}{H_3C{-}CH_2{-}\underset{NH_2}{\underset{|}{CH}}{-}CH_3} \xrightarrow[HCOOH]{HCHO} \underset{3^\circ\ \text{amine}}{H_3C{-}CH_2{-}\underset{(CH_3)_2N}{\underset{|}{CH}}{-}CH_3} \xrightarrow[CH_3OH]{H_2O_2} H_3C{-}CH_2{-}\underset{(CH_3)_2N^+-O^-}{\underset{|}{CH}}{-}CH_3$$

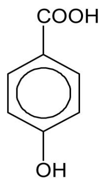
$$\xrightarrow{\Delta} \underset{(major)}{H_3C{-}CH_2{-}CH{=}CH_2} + \underset{(minor)}{H_3C{-}CH_2{=}CH{-}CH_3}$$

Hence the answer is (A).

38. The correct order of decreasing acid strength of following carboxylic acids is :


I


II


III

(A) II > III > I (B) I > II > III (C) III > II > I (D) III > I > II

Solution :

Due to ortho effect.

Hence the answer is (B).

39. The reaction, $\frac{1}{2}H_2(g) + AgCl(s) \longrightarrow H^+(aq) + Cl^-(aq) + Ag(s)$ occurs in the galvanic cell :

(A) Ag/AgCl(s)/KCl(solution)/$AgNO_3$ (solution)/Ag

(B) Pt/H_2(g)/HCl(solution)/$AgNO_3$(solution)/Ag

(C) Pt/H_2(g)/HCl(solution)/AgCl(s)/Ag

(D) Pt/H_2(g)/KCl(solution)/AgCl(s)/Ag

Solution :

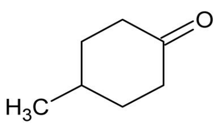
Construction of galvanic cell.

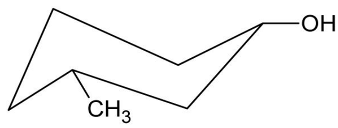
Hence the answer is (C).

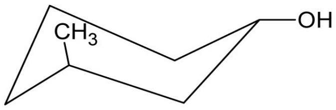
40. $\left(^{\ominus}OOC{-}CH_2CH_2\underset{CH_3}{\underset{|}{CH}}{-}CH_2CH_2COO^{\ominus}\right)$

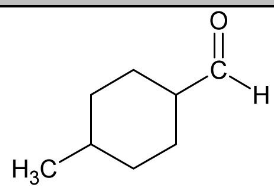
$Ca^{2+} \xrightarrow{Heat} (A) \xrightarrow[(ii)\ H_2O^+]{(i)\ LiAlH_4} (B) + (C)$

What is major product B ?

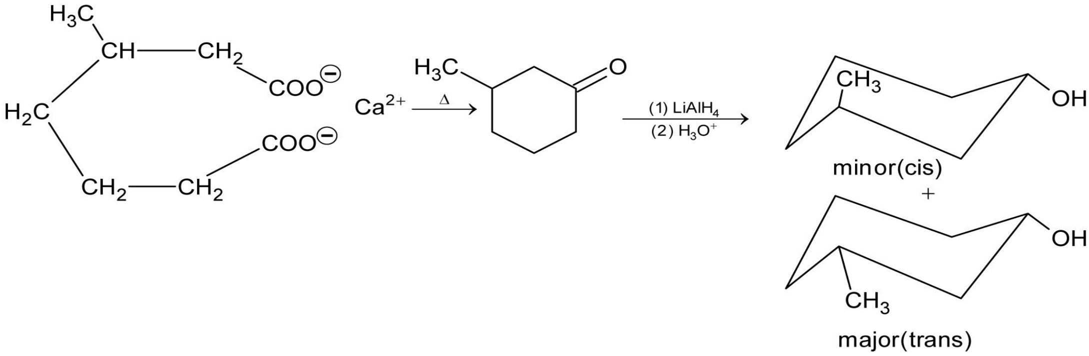
(A) 

(B) 

(C) 

(D) 

Solution :



Hence the answer is (B).

41. A vessel contains $H_2(g)$ and $H_2S(g)$ at 2 atm and 4 atm respectively at 1000 K and the mixture is allowed to attain equilibrium at 1000 K

$8H_2S(g) \rightleftharpoons 8H_2(g) + S_8(s)$

At equilibrium, $\left(\dfrac{n_{H_2}}{n_{H_2S}}\right)_{eqb} = \left(\dfrac{n_{H_2S}}{n_{H_2}}\right)_{initial}$

What is the correct statement

(A) Kp = Kc (RT) (B) Kc = 2.56 (C) Kp = Kc $(RT)^8$ (D) Kc = 256

Solution :

$$\left[\frac{\text{moles of } H_2/V}{\text{moles of } H_2S/V}\right]^8 = \left(\frac{P_{H_2}}{P_{H_2S}}\right)^8 = 2^8 = 256$$

Hence the answer is (D).

42. In a solid AB having fcc structure, A atom occupy the corners of the cubic unit cell. If all the face- centred atoms along one of the axis are removed, then the resultant stoichiometry of the solid is :

(A) AB_2 (B) A_2B (C) A_4B_3 (D) A_3B_4

Solution :

If we remove face-centred atom of one axis, two face atoms are removed.
Thus A is at 8 corners and B at 4 faces

$A = \frac{8}{8} = 1$, $B = \frac{4}{2} = 2$

$\therefore AB_2$

Hence the answer is (A).

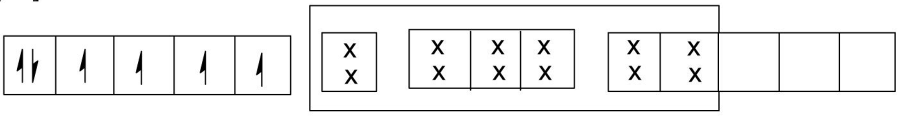
43. Select correct statement for $[Fe(H_2O)_6]SO_4$ is

(A) Diamagnetic and d^2sp^3 (B) Diamagnetic and = 0 B.M.

(C) Paramagentic and outer d - complex (D) Paramagentic and $\mu = \sqrt{8}$ B.M.

Solution :

$Fe^{2+} \longrightarrow [Ar]$ $3d^6$ $4s^0$ $4p^0$ $4d^0$



sp^3d^2 hybridisation

$\mu = \sqrt{4(4+2)} = \sqrt{24}$ B.M.

Hence the answer is (C).

44. $\underset{\text{Greenish crystalline solid}}{A} + BaCl_2 \longrightarrow \underset{\text{white ppt. insoluble in HCl}}{B}$

$A \xrightarrow{\Delta} H_2O + C + \underset{(Gas)}{D} + \underset{(Gas)}{E}$

Identify B and D respectively

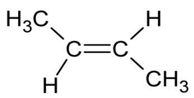
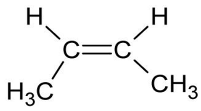
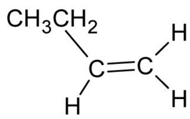
(A) SO_2, SO_3 (B) $BaSO_4$, Fe_2O_3 (C) $BaSO_4$, SO_2 (D) $BaSO_4$, SO_3

Solution :

$A \rightarrow FeSO_4.7H_2O$, $B \rightarrow BaSO_4$, $C \rightarrow Fe_2O_3$, $D \rightarrow SO_2$, $E \rightarrow SO_3$

Hence the answer is (B).

45. $CH_3-CH_2-\underset{}{\overset{Br}{\overset{|}{C}}}H-CH_3 \xrightarrow[C_2H_5OH]{C_2H_5ONa} \underset{\text{major product}}{(A)}$

(A)  (B)  (C)  (D) $CH_3CH_2\overset{OC_2H_5}{\overset{|}{C}}H-CH_3$

Solution :

$CH_3CH_2\overset{Br}{\overset{|}{C}}H-CH_3 \xrightarrow[C_2H_5OH]{C_2H_5ONa} \underset{\text{(cis \& trans)}}{CH_3CH_2=CH-CH_3}$

Trans is major product.

Hence the answer is (A).

Integer Type

46. An aqueous solution contains 10% ammonia by mass and has a density of 0.99 g/cm^3. What is the pH of this solution? (Ka for NH_4^+ = 5.0 × 10^{-10}M)

Solution :

As given

$$\frac{\text{mass of } NH_3}{\text{mass of solution}} = \frac{10}{100}$$

$$\therefore\ M(NH_3) = \frac{10 \times 1000 \times 0.99}{100 \times 17} = 5.82$$

$$NH_3 + H_2O \longrightarrow NH_4OH$$

$$\begin{array}{lccc} & NH_4OH & \rightleftharpoons NH_4^+ & + OH^- \\ t = 0 & C & 0 & 0 \\ \text{at eq}^m & C - C\alpha & C\alpha & C\alpha \end{array}$$

$$\left[OH^-\right] = C\alpha = C.\sqrt{\frac{K_b}{C}}$$

$$K_b = \frac{K_w}{K_a} = \frac{10^{-14}}{5 \times 10^{-10}} = 2 \times 10^{-5}$$

$$\therefore\ \left[OH^-\right] = \sqrt{2 \times 10^{-5} \times 5.82} = 1.07 \times 10^{-2} M$$

$$\therefore\ \left[H^+\right] = \frac{10^{-14}}{1.07 \times 10^{-2}} = 0.93 \times 10^{-12} M$$

pH = 12

Hence the answer is (12.0).

47. One mole of a mixture of CO and CO_2 requires exactly 20 gm of NaOH in solution for complete conversion of all the CO_2 into sodium carbonate. How much NaOH in grams would it require for conversion into Na_2CO_3, if the mixture (one mole) is completely oxidised to the CO_2 ?

Solution :

$$2NaOH + CO_2 \longrightarrow Na_2CO_3 + H_2O$$

$$\because\ \frac{1}{2} \times \text{moles of NaOH used} = \text{moles of } CO_2$$

$$\therefore\ \text{moles of } CO_2 \text{ in mixture} = \frac{1}{2} \times \frac{20}{40} = \frac{1}{4}$$

Mole of CO in mixture = $1 - \frac{1}{4} = \frac{3}{4}$

If this CO is completely oxidised to CO_2 then moles of CO_2 formed = $\frac{3}{4}$

Total moles of CO_2 = $\frac{1}{4} + \frac{3}{4} = 1$

Moles of NaOH required = 2 × moles of CO_2 = 2 × 1 = 2 moles

∴ mass of NaOH required = 2 × 40 = 80 gm

Hence the answer is (80).

48. A Zn rod weighing 25 g was kept in 100 ml of 1 (M) $CuSO_4$ solution. After a certain time the molarity of Cu^{2+} in solution was 0.8. What was the mass of Zn rod in grams after cleaning ? (At. Wt. of Zn = 65.4, Cu = 60.0)

Solution :

$$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$$

$\because$ meq. $= N \times V(ml)$

meq. of Cu^{2+} lost = meq. of Cu^{2+} before reaction – meq. of Cu^{2+} after reaction.

= $(1000 \times 1 \times 2) - (100 \times 0.8 \times 2) = 40$

$\therefore$ meq. of Cu^{2+} lost = meq. of Zn lost = 40

$$\frac{W}{E} \times 1000 = \frac{W}{65.4/2} \times 1000 = 40 \Rightarrow W = 1.3gm$$

$\therefore$ Net mass of Zn rod = 25 – 1.3 = 23.7 gm

Hence the answer is (23.7).

49. For the hydrolysis of esters in alkaline medium rate expression is $-\frac{d[ester]}{dt} = k[ester][alkali]$ when excess alkali is used, then the overall order of the reaction is

Solution :

Hence the answer is (1).

50. Calculate the value of molal elevation constant for water in K/molality, if ΔS for vaporization is 26.33 cal K^{-1} mol^{-1} for water.

Solution :

$$K_b = \frac{RT_b.M}{1000\Delta S}, \ \Delta S = \frac{\Delta H}{T}$$

$$= \frac{2 \times 373 \times 18}{100 \times 26.33} = 0.51$$

Hence the answer is (0.51).

❒ ❒ ❒ ❒ ❒ ❒ ❒

51. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)$ and $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2 (C) $\frac{b}{a}$ (D) $\sqrt{\frac{c}{a}}$

Solution :

Let $A(\alpha, 0)$ and $B(\beta, 0)$ be the two points

$OT^2 = OA \cdot OB = \alpha\beta = \frac{c}{a}$

Hence the answer is (D).

52. The value of $\lim_{x\to 0} \frac{\ln\left(1+\sin^3 x\cos^2 x\right)\cot\left(\ln^3(1+x)\right)\tan^4 x}{\sin\left(\sqrt{x^2+2}-\sqrt{2}\right)\ln\left(1+x^2\right)}$

(A) 1 (B) $2\sqrt{2}$ (C) $\frac{1}{2\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}$

Solution :

Hence the answer is (B).

53. The solution of differential equation

$yy'' - 2y'\cdot y' = 0$, which passes through the point $x = 1$, $y = 1$

(A) $y = \frac{1}{A(x-1)+1}$ (B) $y = \frac{x}{-A(x-1)+1}$ (C) $y = \frac{1}{-A(x-1)+1}$ (D) none of these

Solution :

$\frac{y''}{y'} = \frac{2y'}{y}$, on integrating, we get $\ln y' = 2\ln y + c$ so $y' = Ay^2$

Again integration, we get $-\frac{1}{y} = Ax + B$

Put $x = 1$, $y = 1 \Rightarrow \frac{1}{y} = -A(x-1)+1$

Hence the answer is (C).

54. In a regular hexagon ABCDEF, $\overline{AE}$ is equal to

(A) $\overline{AC}+\overline{AF}+\overline{AB}$ (B) $\overline{AC}+\overline{AF}-\overline{AB}$ (C) $\overline{AC}+\overline{AB}-\overline{AF}$ (D) none of these

Solution :

$\overline{AE} = \overline{AC}+\overline{CD}+\overline{DE} = AC + AF - AB$

$\because \left(\overline{DE} = -\overline{AB}\right)$ and $\left(\overline{CD} = \overline{AF}\right)$

Hence the answer is (B).

55. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are three non-coplanar vectors such that $\bar{a}+\bar{b}+\bar{c}=\alpha\bar{d}$ and $\bar{b}+\bar{c}+\bar{d}=\beta\bar{a}$, then $\bar{a}+\bar{b}+\bar{c}+\bar{d}$ is equal to

(A) 0 (B) $\alpha\bar{a}$ (C) $\beta\bar{b}$ (D) $(\alpha+\beta)\bar{c}$

Solution :

$\because\ \bar{a}+\bar{b}+\bar{c}+\bar{d}=(\alpha+1)\bar{d}$ and $\bar{a}+\bar{b}+\bar{c}+\bar{d}=(\beta+1)\bar{a}$

$\Rightarrow\ \bar{d}=\left(\frac{\beta+1}{\alpha+1}\right)\bar{a}$ if $(\alpha\neq-1)$

$\Rightarrow\ \left(1-\frac{\alpha(\beta+1)}{\alpha+1}\right)\bar{a}+\bar{b}+\bar{c}=0$

Which is a contradiction

$\because\ \bar{a},\bar{b},\bar{c}$ are non-coplanar

$\therefore\ \alpha=-1,\ \bar{a}+\bar{b}+\bar{c}+\bar{d}=0$

Hence the answer is (A).

56. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ be 3 vectors having magnitude 1, 1 and 2 respectively. If $\bar{a}\times(\bar{a}\times\bar{c})+\bar{b}=0$, the acute angle between $\bar{a}$ and $\bar{c}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Solution :

$(\bar{a}\cdot\bar{c})\bar{a}-(\bar{a}\cdot\bar{a})\bar{c}=-\bar{b}$

$\Rightarrow\ (2\cos\theta\bar{a}-\bar{c})^2=1\ \ (\bar{a}\cdot\bar{c}=(1)(2)\cos\theta)$

$\Rightarrow\ \cos^2\theta=\frac{3}{4}$

$\Rightarrow\ \theta=\frac{\pi}{6}$

Hence the answer is (A).

57. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are any 3 non zero vectors, then the component of $\bar{a}\times(\bar{b}\times\bar{c})$ perpendicular to $\bar{b}$ is

(A) $\bar{a}\times(\bar{b}\times\bar{c})+\left(\frac{(\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{a})}{|b|^2}\right)\bar{b}$

(B) $\bar{a}\times(\bar{b}\times\bar{c})+\left(\frac{(\bar{a}\times\bar{c})\cdot(\bar{a}\times\bar{b})}{|b|^2}\right)\bar{b}$

(C) $\bar{a}\times(\bar{b}\times\bar{c})+\left(\frac{(\bar{b}\times\bar{c})\cdot(\bar{b}\times\bar{a})}{|b|^2}\right)\bar{b}$

(D) $\bar{a}\times(\bar{b}\times\bar{c})+\left(\frac{(\bar{a}\times\bar{b})\cdot(\bar{b}\times\bar{c})}{|b|^2}\right)\bar{b}$

Solution :

Component of $\bar{a}\times(\bar{b}\times\bar{c})$ along $\bar{b}$ is $\left(\left(\frac{(\bar{a}\cdot\bar{c})\bar{b}-(\bar{a}\cdot\bar{b})\bar{c}}{|b|^2}\right)\cdot\bar{b}\right)\bar{b}$

So component of $\bar{a}\times(\bar{b}\times\bar{c})$ perpendicular to $\bar{b}$ is $\bar{a}\times(\bar{b}\times\bar{c})-\left(\frac{(\bar{a}\cdot\bar{b})(\bar{b}\cdot\bar{c})-(\bar{a}\cdot\bar{c})(\bar{b}\cdot\bar{b})}{|b|^2}\right)\bar{b}$

Hence the answer is (D).

58. If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha}=\lambda$, then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is

(A) $\frac{1}{\lambda}$ (B) λ (C) $1-\lambda$ (D) $1+\lambda$

Solution :

$$\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}\times\frac{(1+\sin\alpha+\cos\alpha)}{(1+\sin\alpha+\cos\alpha)}=\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha}$$

Hence the answer is (B).

59. Let n be an odd integer. If $\sin(n\theta)=\sum_{r=0}^{n}b_r\sin^r\theta$ for all real θ, then

(A) $b_0 = 1,\ b_1 = 4$ (B) $b_0 = -1,\ b_1 = n^2 - 4n + 2$

(C) $b_0 = 0,\ b_1 = n$ (D) $b_0 = 0,\ b_1 = n^2 - 4n + 2$

Solution :

$\sin(n\theta) = b_0 + b_1 \sin\theta + b_2 \sin^2\theta$ ….. $b_n \sin^n\theta$

Given n is an odd integer, put $\theta = 0$ to get $b_0 = 0$ and after differentiation w.r.t. θ, and putting θ = 0, we get $b_1 = n$

Hence the answer is (C).

60. If the area of a ΔABC be λ, then

$a^2 \sin 2B + b^2 \sin 2A$ is

(A) 2λ (B) 3λ (C) 4λ (D) none of these

Solution :

$a^2 \sin 2B + b^2 \sin 2A = 2a^2 \sin B \cos B + 2b^2 \sin A \cos A$

$\left(\because \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{1}{2R}\right)$

$$\Rightarrow \frac{ab}{R}(a\cos B+b\cos A)=\frac{abc}{R}=2bc\sin A = 4\left(\frac{1}{2}bc\sin A\right)=4\lambda$$

Hence the answer is (C).

61. The lengths of sides of a triangle are $a-b$, $a+b$ and $\sqrt{3a^2+b^2}$ $(a > b > 0)$. The sine of its largest angle is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

Solution :

Let $p = a - b$, $q = a + b$, $r=\sqrt{3a^2+b^2}$

Greatest angle θ, $\cos\theta=\frac{p^2+q^2-r^2}{2pq}=-\frac{1}{2}$, $\theta=\frac{2\pi}{3}$

$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$

Hence the answer is (C).

62. In a right angle triangle ABC, D is a mid-point of hypotenuse AB and E is the mid-point of AC. Segments BE and CD intersect at F. If $AC=\sqrt{2}$ and BC = 1, then cos $\angle$BFC is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) none of these

Solution :

Slope of CD = $\dfrac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \dfrac{1}{\sqrt{2}}$, slope of BE = $-\sqrt{2}$

$\Rightarrow$ CD $\perp$ BE $\Rightarrow \cos\angle BFC = 0$

Hence the answer is (D).

63. A ray of light incident at the point (–2, –1) get reflected from the tangent at (0, –1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moves, is

(A) $4x - 3y + 11 = 0$ (B) $4x + 3y + 11 = 0$

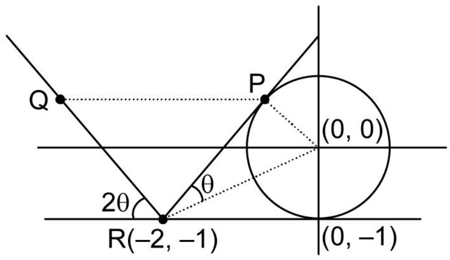
(C) $3x + 4y + 11 = 0$ (D) $4x + 3y + 7 = 0$

Solution :

$$\tan 2\theta = \frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{4}{3}$$

Slope of QR $= -\dfrac{4}{3}$

$\Rightarrow$ Equation of incident ray QR is $3y + 4x + 11 = 0$



Hence the answer is (B).

64. A variable plane forms a tetrahedron constant volume $64k^3$ with the co-ordinate planes and the origin, then locus of the centroid of the tetrahedron is

(A) $x^3 + y^3 + z^3 = 6k^3$ (B) $xyz = 6k^3$

(C) $x^2 + y^2 + z^2 = 4k^2$ (D) $x^{-2} + y^{-2} + z^{-2} = 4k^{-2}$

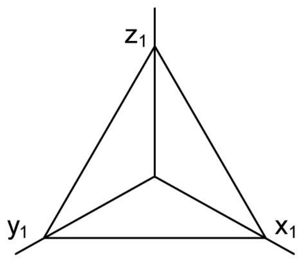
Solution :

$\dfrac{x_1}{4} = \mu$, $\dfrac{y_1}{4} = \gamma$, $\dfrac{z_1}{4} = \omega$

$x_1 = 4\mu$, $y_1 = 4\gamma$, $z_1 = 4\omega$

$$v = \frac{1}{6}\begin{vmatrix} 4\mu & 0 & 0 \\ 0 & 4\gamma & 0 \\ 0 & 0 & 4\omega \end{vmatrix}$$

$\Rightarrow xyz = 6k^3$



Hence the answer is (B).

65. The line $\dfrac{x-2}{2} = \dfrac{y+1}{2} = \dfrac{z-1}{-1}$ intersects the curve $xy = c^2$, in x – y plane, if c is equal to

(A) ± 1 (B) $\pm\dfrac{1}{3}$ (C) ± 2 (D) none of these

Solution :

Put z = 0 in the given line

$\Rightarrow$ x = 4 and y = 1

Hence the answer is (C).

66. A circle of radius r passes through both foci of and exactly four points on, the ellipse with equation $x^2 + 16y^2 = 16$. The set of all possible values of r is an interval [a, b). What is a + b ?

(A) $5\sqrt{2}+4$ (B) $\sqrt{17}+7$ (C) $6\sqrt{2}+3$ (D) $\sqrt{15}+8$

Solution :

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Hence the answer is (D).

67. Maximum sum of coefficients in the expansion of $(1 - x\sin\theta + x^2)^n$ is

(A) 1 (B) 2^n (C) 3^n (D) 0

Solution :

Put x = 1

$\Rightarrow (1 - \sin\theta + 1)^n$

So maximum value occurs when $\sin\theta = -1$, which is 3^n

Hence the answer is (C).

68. The value of $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}$ is

(A) $\sin x - 6\tan^{-1}(\sin x) + c$ (B) $\sin x - 2(\sin x)^{-1} + c$

(C) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$ (D) none of these

Solution :

$$I = \int \frac{\left(\cos^2 x + \cos^4 x\right).\cos x dx}{\sin^2 x + \sin^4 x} = \int \frac{1-t^2+\left(1-t^2\right)^2}{t^2+t^4}dt$$

$$= \int \frac{\left(1-t^2\right)\left(2-t^2\right)}{t^2\left(1+t^2\right)}dt = 2\int \frac{2-t^2}{1+t^2}dt - \int \frac{\left(2-t^2\right)}{t^2}dt$$

$$= 6\int\left(t^2 - \frac{1}{1+t^2}\right)dt - 4\int\frac{dt}{t^2} + \int dt = 2\int\frac{dt}{t^2} - 6\int\frac{dt}{1+t^2} + \int dt$$

$$= \frac{-2}{t} - 6\tan^{-1}(t) + t + c = \sin x - 2(\sin x)^{-1} - 6 \quad \tan^{-1}(\sin x) + c$$

Hence the answer is (C).

69. If $A = \int_1^{\sin\theta} \frac{t\ dt}{1+t^2}$ and $B = \int_1^{\text{cosec}\theta} \frac{dt}{t(1+t^2)}$, then the value of $\begin{vmatrix} A & A^2 & B \\ e^A e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}$ is

(A) $\sin\theta$ (B) $\text{cosec}\ \theta$ (C) 0 (D) 1

Solution :

$$A = \int_1^{\sin\theta} \frac{tdt}{1+t^2}$$

Let $t = \frac{1}{x}$, $dt = -\frac{1}{x^2}dx$, $A = \int_1^{\cos\text{sc}\theta} \frac{1}{x}.\frac{-1}{x^2}.\frac{dx}{\frac{x^2+1}{x^2}} = -\int_1^{\cos\text{ec}\theta} \frac{dx}{x\left(1+x^2\right)} = -B$

$\therefore$ A + B = 0

$$\Delta = \begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} = \begin{vmatrix} A & A^2 & -A \\ 1 & A^2 & -1 \\ 1 & 2A^2 & -1 \end{vmatrix} = 0$$

Hence the answer is (C).

70. If z is a complex number satisfying $|z|^2 - |z| \quad 2 < 0$, then the value of $|z^2 + z \sin\theta|$, for all values of θ, is

(A) equal to 4 (B) equal to 6 (C) more than 6 (D) less than 6

Solution :

$|z|^2 - |z| - 2 < 0$

$\Rightarrow (|z| - 2)\ (|z| + 1) < 0 \Rightarrow |z| < 2$

Now $|z^2 + z \sin\theta| \le |z|^2 + |z \sin\theta| \le |z|^2 + |z| < 4 + 2 = 6$

Hence the answer is (D).

Integer Type

71. Let N be any four digit number say $x_1\ x_2\ x_3\ x_4$. Then maximum value of $\frac{N}{x_1 + x_2 + x_3 + x_4}$ is equal to

Solution :

$$\frac{N}{x_1 + x_2 + x_3 + x_4} = \frac{1000x_1 + 100x_2 + 10x_3 + x_4}{x_1 + x_2 + x_3 + x_4} = 1000 - \frac{(900x_2 + 990x_3 + 999x_4)}{(x_1 + x_2 + x_3 + x_4)}$$

$\Rightarrow$ Maximum value of $\frac{N}{x_1 + x_2 + x_3 + x_4} = 1000$

Hence the answer is (1000).

72. The maximum value of $f(x) = \frac{x^4 - x^2}{x^6 + 2x^3 - 1}$, $x > 1$ is

Solution :

Hence the answer is (0.167).

73. Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process $\frac{a}{b}$, where a and b are relatively prime positive integers. What is a + b ? (For example, he succeeds if his sequence of tosses is H T H H H H H H)

Solution :

For 6 to 8 heads, we are guaranteed to hit 4 heads, so the sum here is

$$\binom{8}{2} + \binom{8}{1} + \binom{8}{0} = 28 + 8 + 1 = 37$$

For 4 heads, you have to hit the 4 heads at the start so there's only one way, 1.

For 5 heads, we either start of with 4 heads, which gives us 4C1 = 4 ways to arrange the other flips, or we start off with five heads and one tail, which has 6 ways minus the 2 overlapping cases, HHHHHTTT and HHHHTHTT. Total ways 8.

Then we sum to get 46. There are a total of $2^8 = 256$ possible sequences of 8 coin flips, so the probability is $\frac{46}{256} = \frac{23}{128}$. Summing, we get 23 + 128 = 151

Hence the answer is (151).

74. $f(x) + f\left(1-\frac{1}{x}\right) = 1 + x$ for $x \in R - \{0, 1\}$. The value of f(2) is equal to

Solution :

Let $y = 1 - \frac{1}{x}$

$f(y) + f\left(1-\frac{1}{y}\right) = f\left(1-\frac{1}{x}\right)+f\left(x-\frac{x}{x-1}\right) = f\left(1-\frac{1}{x}\right)+f\left(\frac{1}{1-x}\right)=2-\frac{1}{x}$ ..... (1)

put $z = \frac{1}{1-x}$

$f(z) + f\left(1-\frac{1}{z}\right) = f\left(\frac{1}{1-x}\right)+f(x)=1+\frac{1}{1-x}$ ..... (2)

subtract

$f(x) - f\left(1-\frac{1}{x}\right)=\frac{1}{1-x}+\frac{1}{x}-1$

$f(x) = \frac{1}{2}\left(\frac{1}{1-x}+\frac{1}{x}+x\right)$.

Alternate:

Putting $x = 2, \frac{1}{2}$ and –1 successively

f(2) + f(1/2) = 3 ..... (1)

f(1/2) + f(–1) = 3/2 ..... (2)

and f(–1) + f(2) = 0 ..... (3)

Solving, we get f(2) = 3/4.

Hence the answer is (0.75).

75. If $f : R \to R$ is a function satisfying the property $f(2x + 3) + f(2x + 7) = 2, \forall x \in R$, then the period of f(x) is

Solution :

f(2x + 3) + f(2x + 7) = 3 ..... (1)

Replace x by x + 1, f(2x + 5) + f(2x + 9) = 2 ..... (2)

Now replace x by x + 2, f(2x + 7) + f(2x + 11) = 2 ..... (3)

from (1) – (3) we get f(2x + 3) – f(2x + 11) = 0

$f(2x + 3) = f(2x + 11) \Rightarrow T = 4$.

Hence the answer is (4).

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