Answer Key

PHYSICS

1.	b	2.	a	3.	b	4.	a	5.	a	6.	a	7.	d	8.	b	9.	b	10.	a
11.	a	12.	a	13.	a	14.	b	15.	a	16.	b	17.	a	18.	d	19.	c	20.	b
21.	30	22.	12.00	23.	3	24.	18	25.	2.45										

CHEMISTRY

1.	d	2.	d	3.	c	4.	a	5.	c	6.	a	7.	c	8.	d	9.	a	10.	d
11.	a	12.	b	13.	c	14.	c	15.	b	16.	c	17.	a	18.	b	19.	a	20.	a
21.	4	22.	3	23.	21	24.	10	25.	5										

MATHEMATICS

1.	b	2.	c	3.	b	4.	a	5.	b	6.	b	7.	b	8.	b	9.	b	10.	d
11.	a	12.	b	13.	b	14.	b	15.	c	16.	b	17.	b	18.	c	19.	d	20.	c
21.	0.25	22.	5	23.	2	24.	0.5	25.	16										

Question-wise Detailed Solution

PHYSICS

1 A capillary tube of radius r is lowered into a liquid of surface tension T and density ρ. The angle of contact between the solid and the free surface of the liquid is $\theta = 0^\circ$. During the process in which the liquid rises in the capillary, the work done by surface tension is

Solution

The liquid rise in capillary tube is :

$h_0 = \frac{2T}{\rho g r}$

The work done by surface tension will be :

$W = F h_0 = (2\pi T r)\left(\frac{2T}{\rho g r}\right) = \frac{4\pi T^2}{\rho g}$

2 Assertion: The phase difference between any two points on a wave front is zero.

Reason: Light from the source reaches every point of the wave front at the same time.

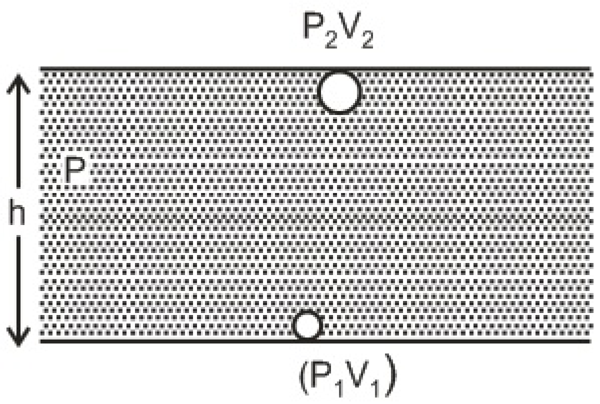
Solution

If both When points receives energy at same time, will oscillate in same phase or identical manner

3 An inverted bell, lying at the bottom of lake $47.6 \mathrm{~m}$ deep, has $50 \mathrm{~cm}^3$ of air trapped in it. The bell is brought to the surface of lake. The volume of the trapped air will become (atmospheric pressure $= 70 \mathrm{~cm}$ of Hg and density of $\mathrm{Hg} = 13.6 \mathrm{g/cm}^3$)

Solution

According to Boyle's law, pressure and volume are inversely proportional to each other i.e. $\mathrm{p} \propto \frac{1}{\mathrm{v}}$



$\Rightarrow \quad \mathrm{P}_1 \mathrm{V}_1 = \mathrm{P}_2 \mathrm{V}_2$

$\Rightarrow \quad (\mathrm{P}_0 + \mathrm{h} \rho_{\mathrm{w}} \mathrm{g}) \mathrm{V}_1 = \mathrm{P}_0 \mathrm{V}_2$

$\Rightarrow \quad \mathrm{V}_2 = \left(1 + \frac{\mathrm{h} \rho_{\mathrm{w}} \mathrm{g}}{\mathrm{P}_0}\right) \mathrm{V}_1$

$V_2 = \left(1 + \frac{47.6 * 1 * 1000 * 10}{70 * 10^{-2} * 13.6 * 1000 * 10}\right) V_1$

$[As\ P_2 = P_0 = 70\ cm\ of\ Hg = 70 * 10^{-2} * 13.6 * 1000 * 10]$

$\Rightarrow \quad \mathrm{V}_2 = (1 + 5) 50 \mathrm{~cm}^3 = 300 \mathrm{~cm}^3$

4 A radioactive sample S_1 having the activity A_1 has twice the number of nuclei as another sample S_2 of activity A_2. If $A_2 = 2A_1$, then the ratio of half-life of S_1 to the half-life of S_2 is

Solution

Activity, $A = \lambda N = \frac{0.693}{T_{1/2}} N$

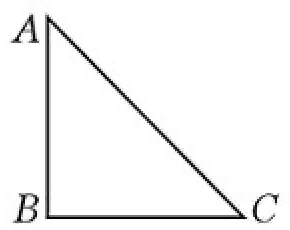
Where $T_{1/2}$ is the half-life of a radioactive sample,

$\therefore \quad \frac{A_1}{A_2} = \frac{N_1}{T_1} \times \frac{T_2}{N_2}$

$\frac{T_1}{T_2} = \frac{A_2}{A_1} \times \frac{N_1}{N_2}$

$= \frac{2A_1}{A_1} \times \frac{2N_2}{N_2} = \frac{4}{1}$

5 Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$, respectively, in the steady-state. Assuming that only heat conduction takes place, the temperature of the point C is

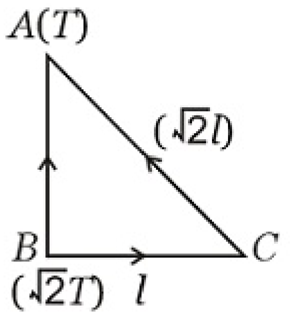


Solution

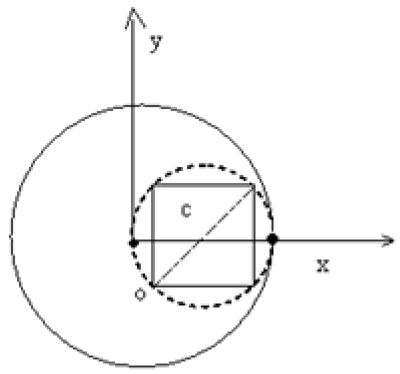
As $T_B > T_A$, heat flows from *B* to *A* through both paths *BA* and *BCA*.
Rate of heat flow in BC = Rate of heat flow in CA

$$\frac{KA\left(\sqrt{2}T-T_c\right)}{l}=\frac{KA(T_c-T)}{\sqrt{2}l}$$

Solving this, we get $T_c=\frac{3T}{\sqrt{2}+1}$



6 There is a thin uniform disc of radius R and mass per unit area σ, in which a hole of radius $\mathrm{R}/2$ has been cut out as shown in the figure. Inside the hole, a square plate of same mass per unit area σ is inserted so that its corners touch the periphery of the hole. The distance of the centre of mass of the system from the origin is



Solution

Side of square $=\mathrm{R}\cos 45^{\circ}=\frac{\mathrm{R}}{\sqrt{2}}$

Area of square $=\frac{\mathrm{R}^2}{2}$

$$\mathrm{X}\,(\mathrm{COM})=\frac{\left(\pi\times\mathrm{R}^2\times\sigma\times 0+\pi\times\frac{\mathrm{R}^2}{4}(-\sigma)\times\frac{\mathrm{R}}{2}+\frac{\mathrm{R}^2}{2}\times\sigma\times\frac{\mathrm{R}}{2}\right)}{\left(\pi\times\mathrm{R}^2\times\sigma+\pi\times\frac{\mathrm{R}^2}{4}(-\sigma)+\frac{\mathrm{R}^2}{2}\times\sigma\right)}$$

$$=\frac{\mathrm{R}(2-\pi)}{2(3\pi+2)}$$

$\therefore$ The centre of mass of the system is at a distance of $\frac{\mathrm{R}(2-\pi)}{2(3\pi+2)}$ from the centre O towards the plate as shown in the figure

7 Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $X_P(t)=at+bt^2$ and $X_Q(t)=ft-t^2$. At what time do the cars have the same velocity?

Solution

$$v_P=\frac{dx_P}{dt}=a+2bt$$

$$v_Q=\frac{dx_Q}{dt}=f-2t$$

$$v_P=v_Q$$

$$\Rightarrow a+2bt=f-2t$$

$$2t+2bt=f-a$$

$$\Rightarrow t=\frac{f-a}{2(b+1)}$$

8 Rain falls with velocity $7\hat{i}-10\hat{j}$ where x-axis is horizontal and y-axis is vertical. With what velocity should a man run so that he sees rain falling vertically.

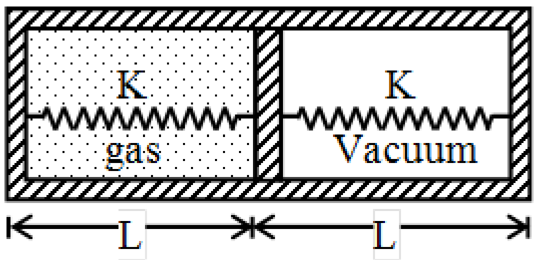
Solution

$\bar{v}_{R/m} = \bar{v}_{R/g} - \bar{v}_{m/g}$

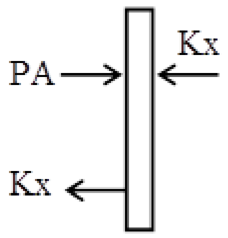
$-10\hat{j} = 7\hat{i} - 10\hat{j} - \bar{v}_{m/g}$

$\bar{v}_{m/g} = 7\hat{i}$

9 Area of piston is 1 m². When heat is supplied to the gas it expands and displaces piston by $\frac{L}{2}$ where L = 1 m. Natural length of springs is L = 1m. Spring constant K = 100 N/m. The pressure of gas in final situation is – (considering equilibrium)



Solution



$P = \frac{2Kx}{A}$

$= \frac{2 \times 100}{1} \times \frac{1}{2}$

$= 100\ N/m^2$

10 The primary winding of a transformer has 100 turns and its secondary winding has 200 turns. The primary is connected to an AC supply of 120 V and the current flowing in it is 10 A. The voltage and the current in the secondary are

Solution

$\frac{E_s}{E_p} = \frac{n_s}{n_p}$ *or* $E_s = E_p \times \left(\frac{n_s}{n_p}\right)$

$\therefore E_s = 120 \times \left(\frac{200}{100}\right) = 240\ V$

$\frac{I_p}{I_s} = \frac{n_s}{n_p}$ *or* $I_s = I_p \left(\frac{n_p}{n_s}\right)$

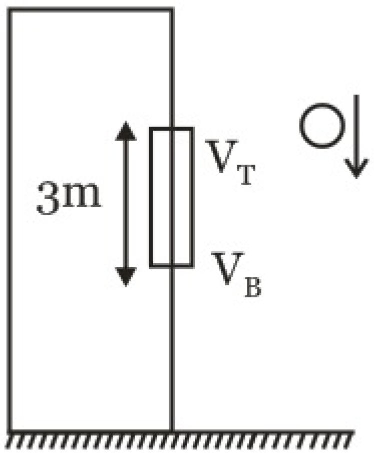
$\therefore I_s = 10 \left(\frac{100}{200}\right) = 5\ A$

11 A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m length of a window some distance from the top of the building. If the speed of the ball at the top and at the bottom of the window are v_T and v_B respectively, then (take g = 10 m/s²) :

Solution

$s = \frac{(u+v)}{2} t$

$3 = \frac{(v_T + v_B)}{2} \times 0.5$



$v_T + v_B = 12 m/s$

Also $v_B = v_T + (9.8)(0.5)$

$v_B - v_T = 4.9 m/s$

12 A deflection magnetometer is adjusted in the usual way. When a magnet is introduced, the deflection observed is θ, and the period of oscillation of the needle in the magnetometer is T. When the magnet is removed, the period of oscillation is T_0. The relation between T and T_0 is

Solution

For first case

$T = 2\pi \sqrt{\frac{I}{M\sqrt{F^2+H^2}}} \qquad \ldots(i)$

When magnet is removed

$T_0 = 2\pi \sqrt{\frac{I}{MH}} \qquad \ldots(ii)$

Also, $\frac{F}{H} = \tan\ \theta$

From Eqs. (i) and (ii) we have

$\frac{T}{T_0} = \sqrt{\frac{H}{\sqrt{F^2+H^2}}}$

$\frac{T^2}{T_0^2} = \cos\ \theta\, T^2 = T_0^2 \cos\ \theta$

13 How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?

Solution

From the nth state, the atom may go to (n - 1)th state, ..., 2nd state of 1st state, So there are (n - 1) possible transitions starting from the nth state. The atoms reaching (n - 1)th state may make (n - 2) different transitions. Similarly for other lower sates. The total number of possible transitions is

= (n - 1) + (n - 2) + (n - 3) + ... 2 + 1

$= \frac{n(n-1)}{2}$

14 Two gold pieces, each of mass 0.035 g, are placed in a box of mass 2.3 g. The total mass of the box with gold pieces is

Solution

$(2.3 + 0.035 + 0.035)\ \mathrm{g} = 2.37\ \mathrm{g}$

But we have to retain only one decimal place.

So, the total mass is $2.4\ \mathrm{g}$

15 An electron having kinetic energy $10\ \mathrm{eV}$ is circulating in a path of radius $0.1\ \mathrm{m}$ in an external magnetic field of intensity $10^{-4}\ \mathrm{T}$. The speed of the electron will be

Solution

When charge enters into a perpendicular magnetic field, it starts to move in a circular path.
Radius of a circular path followed by a charge particle:
$r = \frac{mV}{qB}$
$V = 1.76 \times \left(10^{6}\right)\ \mathrm{ms}^{-1}$

16 A screen is placed $90\ \mathrm{cm}$ from an object. The image of an object on the screen is formed by a convex lens at two different locations separated by $20\ \mathrm{cm}$. The focal length of the lens is

Solution

From displacement method

$\mathrm{D} = 30\ \mathrm{cm}$ and $\mathrm{d} = 20\ \mathrm{cm}$

$\mathrm{f} = \frac{\mathrm{D}^2 - \mathrm{d}^2}{4\mathrm{D}} \Rightarrow \mathrm{f} = \frac{90^2 - 20^2}{4 \times 90}$

$\mathrm{f} = \frac{110 \times 70}{4 \times 90} \Rightarrow \mathrm{f} = 21.4\ \mathrm{cm}$

17 The magnitude of the X and Y components of $\overrightarrow{\mathrm{A}}$ are 7 and 6. Also the magnitudes of X and Y components of $\overrightarrow{\mathrm{A}} + \overrightarrow{\mathrm{B}}$ are 11 and 9 respectively. What is the magnitude of $\overrightarrow{\mathrm{B}}$?

Solution

Let $\overrightarrow{\mathrm{A}} + \overrightarrow{\mathrm{B}} = \overrightarrow{\mathrm{R}}$. Given A_x=7 and $A_y = 6$

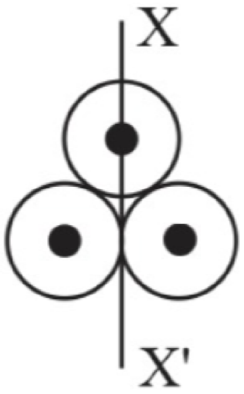
Also $R_x = 11$ and R_y=9. Therefore,

$B_x = R_x - A_x = 11 - 7 = 4$

and $B_y = R_y - A_y = 9 - 6 = 3$

Hence, $B = \sqrt{B_x^2 + B_y^2} = \sqrt{4^2 + 3^2} = 5$

18 Three identical spherical shells, each of mass m and radius r are placed as shown in the figure. Consider an axis XX' which is touching the two shells and passing through diameter of the third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is:



Solution

Total MI of the system,

$$\mathrm{I} = \mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3$$

$$\mathrm{I}_2 = \mathrm{I}_3 = \frac{2}{3}\mathrm{mr}^2 + \mathrm{mr}^2 = \frac{5\mathrm{mr}^2}{3}$$

$$\mathrm{I}_1 = \frac{2}{3}\mathrm{mr}^2$$

$$\therefore \mathrm{I} = 2 \times 5\frac{\mathrm{mr}^2}{3} + \frac{2}{3}\mathrm{mr}^2$$

$$= \frac{12\mathrm{mr}^2}{3} = 4\ \mathrm{mr}^2$$

19 A pendulum has angular amplitude θ. Tension in the string at extreme position is T_1 and at bottom is T_2.

If $T_2 = 2T_1$, then θ is equal to

Solution

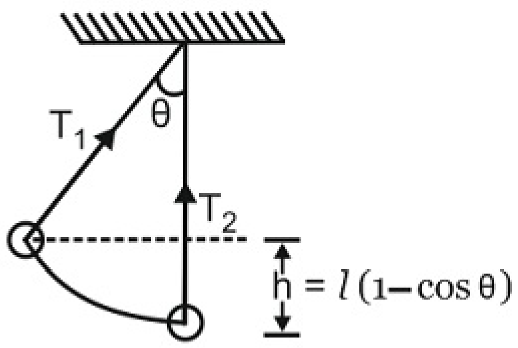
$\mathrm{T}_1 = \mathrm{mg}\ \cos\theta$ ..... (1)

$$\mathrm{T}_2 - \mathrm{mg} = \frac{\mathrm{mv}^2}{l} = \frac{\mathrm{m}}{l}\,(2\mathrm{gh})$$

$$= \frac{2\mathrm{mg}}{l}\,(1 - \cos\theta)l$$

$$= 2\mathrm{mg}\,(1 - \cos\theta)l$$

or $\mathrm{T}_2 = \mathrm{mg} + 2\mathrm{mg}\,(1 - \cos\theta)$ .... (2)



Given $T_2 = 2T_1$

or $\mathrm{mg} + 2\mathrm{mg}\,(1 - \cos\theta) = 2\mathrm{mg}\ \cos\theta$

or $\cos\theta = \frac{3}{4}$

$$\therefore\ \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

20 A shunt of resistance $\left(\frac{1}{m}\right)^{th}$ of the resistance of the galvanometer is used to convert it into an ammeter. The range of the ammeter becomes.

Solution

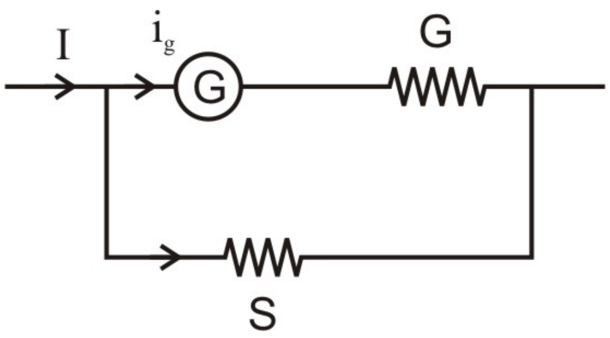
Resistance of shunt is given by : $S = \frac{g}{n-1}$ where $n = \frac{I}{i_g}$

$\frac{g}{m} = \frac{g}{n-1}$

$\therefore n = m + 1$

$\frac{I}{i_g} = m + 1$

$\therefore I = (m+1) i_g$



$(I - i_g) = (n-1) i_g$

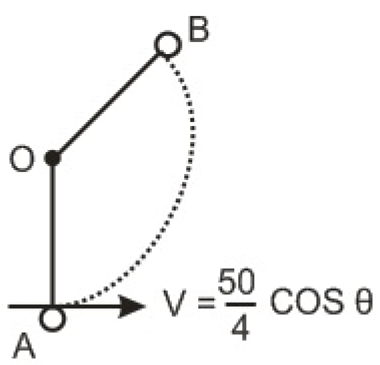
21 A bullet of mass M is fired with a velocity 50 m/s at an angle θ with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3 m and gets embedded in the bob. After the collision the string moves through an angle of 120°. Find the angle θ.
(Take g = 10 m/s²)

Solution

At the highest point, velocity of bullet is 50 cos θ. So, by conservation of linear momentum

M (50 cos θ) = 4Mν

$\nu = \left(\frac{50}{4}\right) \cos\theta$ ..... (i)



At point B, T = 0 but $\nu \neq 0$

Hence, $4\,\mathrm{Mg} \cos 60^\circ = \frac{(4\,\mathrm{M})\,\nu^2}{l}$

or $\nu^2 = \frac{g}{2} l = \frac{50}{3}$ ... (ii)

(as $l = \frac{10}{3}\mathrm{m}$ and $g = 10\ \mathrm{m/s^2}$)

Also, $\nu^2 = u^2 - 2gh$

$= u^2 - 2g\left(\frac{3}{2} l\right)$

$= u^2 - 3(10)\left(\frac{10}{3}\right)$

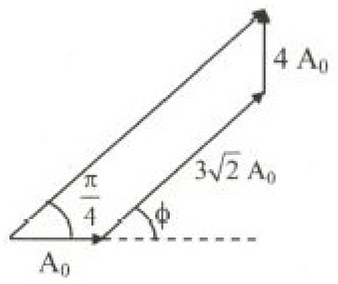
or $\quad \nu^2 = u^2 - 100$

Solving Eqs. (i), (ii) and (iii), we get

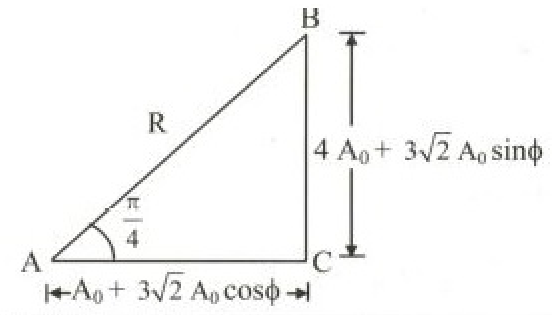
$\cos\theta = 0.86$ or $\theta \approx 30^{\circ}$

22 The equations of three waves are given by $y_1 = A_0 \sin(kx - \omega t)$, $y_2 = 3\sqrt{2}A_0 \sin(kx - \omega t + \phi)$ and $y_3 = 4A_0 \cos(kx - \omega t)$. Theses waves are in the same direction and are superimposed. The phase difference between the resultant-wave and the first wave is $\frac{\pi}{4}$ and $\phi = \frac{\pi}{n} \le \frac{\pi}{2}$, then what is the value of n?

Solution



$\tan\left(\frac{\pi}{4}\right) = \frac{BC}{AC} = \frac{A_0\left(4+3\sqrt{2}\sin\phi\right)}{A_0\left(1+3\sqrt{2}\cos\phi\right)}$



$\Rightarrow \cos\phi - \sin\phi = \frac{1}{\sqrt{2}}$
Squaring both sides, $\Rightarrow \cos^2\phi + \sin^2\phi - 2\cos\phi\sin\phi = \frac{1}{2}$
$\Rightarrow 2\sin\phi\cos\phi = \frac{1}{2}$
$\Rightarrow \sin 2\phi = \frac{1}{2}$
$\Rightarrow \phi = \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{12}$

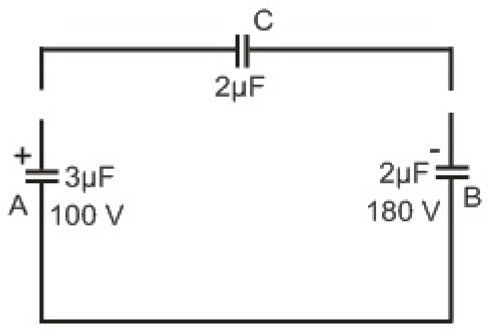
23 A particle of mass m is subjected to an attractive central force of magnitude $6(1\mathrm{Fe}, 1\mathrm{W} \text{ and } 4\mathrm{O})\frac{\mathrm{k}}{\mathrm{r}^2}$, k being a constant. If at instant when the particle is at an extreme position in its closed orbit, at a distance 'a' from the centre of force, its speed is $\sqrt{\left(\frac{\mathrm{k}}{2\mathrm{ma}}\right)}$, if the distance of other extreme position is b. The ratio of a/b is

Solution

$\mathrm{F} = -\frac{\mathrm{k}}{\mathrm{r}^2}; \mathrm{U} = -\int \mathrm{F}\mathrm{dr} = \int \frac{\mathrm{k}}{\mathrm{r}^2}\mathrm{dr} = -\frac{\mathrm{k}}{\mathrm{r}}$
From conservation of momentum (angular) $\mathrm{mv_1a} = \mathrm{mv_2b}$
$\Rightarrow \mathrm{v_2} = \frac{\mathrm{a}}{\mathrm{b}}\mathrm{v_1} = \frac{\mathrm{a}}{\mathrm{b}}\sqrt{\frac{\mathrm{k}}{2\mathrm{ma}}}$
Now, from law of conservation of energy, $\mathrm{K_1} + \mathrm{U_1} = \mathrm{K_2} + \mathrm{U_2}$
$\Rightarrow \frac{1}{2}\mathrm{mv_1^2} - \frac{\mathrm{k}}{\mathrm{a}} = \frac{1}{2}\mathrm{mv_2^2} - \frac{\mathrm{k}}{\mathrm{b}}$
$\Rightarrow \frac{1}{2}\mathrm{m}\left(\frac{\mathrm{k}}{2\mathrm{ma}}\right) - \left(\frac{\mathrm{k}}{\mathrm{a}}\right) = \frac{1}{2}\mathrm{m}\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^2\left(\frac{\mathrm{k}}{2\mathrm{ma}}\right) - \frac{\mathrm{k}}{\mathrm{b}}$
$\Rightarrow \frac{-3\mathrm{k}}{4\mathrm{a}} = \frac{\mathrm{ak}}{4\mathrm{b}^2} - \frac{\mathrm{k}}{\mathrm{b}}$
$\Rightarrow \mathrm{b}^2 - \frac{4\mathrm{a}}{3}\mathrm{b} + \frac{\mathrm{a}^2}{3} = 0$
Solving $\mathrm{b} = \frac{\frac{4\mathrm{a}}{3} - \sqrt{\left(\frac{4\mathrm{a}}{3}\right)^2 - 4\left(\frac{\mathrm{a}^2}{3}\right)}}{2}$
$\therefore \quad \mathrm{b} = \frac{\frac{4\mathrm{a}}{3} - \frac{2\mathrm{a}}{3}}{2} = \frac{\mathrm{a}}{3}$
$\therefore \quad \frac{\mathrm{a}}{\mathrm{b}} = 3$

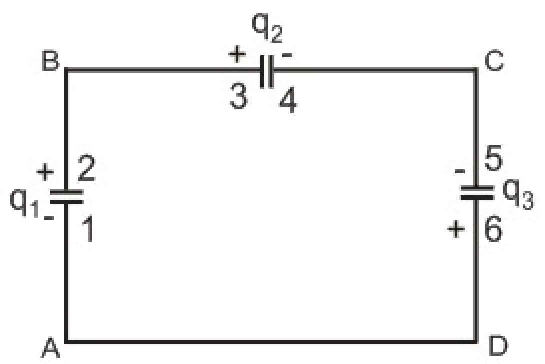
24 Two capacitors A and B with capacities $3\mu\mathrm{F}$ and $2\mu\mathrm{F}$ are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is

positive and that of B is negative. An uncharged 2μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate the amount of electrostatic energy stored in the system after completion of the circuit.



Solution

Charge on capacitor A, before joining with an uncharged capacitor
$q_A = CV = (100)\ (3)\ \mu C = 300\mu C$ Similarly, charge on capacitor B
$q_B = (180)\ (2)\mu C = 360\mu C$



Let q_1, q_2 and q_3 be the charges on the three capacitors after joining them as shown in figure.

(q_1, q_2 and q_3 are in microcoulombs)
From conservation of charge

Net charge on plates 2 and 3 before joining

= net charge after joining $30 = q_1 + q_2$ ... (i)

Similarly, net charge on plates 4 and 5 before joining
= net charge after joining

$-360 = -q_2 - q_3$ or $360 = q_2 + q_3$ ... (ii)

Applying Kirchhoff's second law in closed loop ABCDA

$\frac{q_1}{3} - \frac{q_2}{2} + \frac{q_3}{2} = 0$

or $2q_1 - 3q_2 + 3q_3 = 0$ ... (iii)

Solving Eqs. (i), (ii) and (iii), we get

$q_1 = 90\mu C$, $q_2 = 210\mu C$

and $q_3 = 150\mu C$

Electrostatic energy stored after, completing the circuit

$U_f = \frac{1}{2}\frac{(90\times10^{-6})^2}{(3\times10^{-6})} + \frac{1}{2}\frac{(210\times10^{-6})^2}{(2\times10^{-6})}$

$= 18 \times 10^{-2}$ J

or $U_f = 18$ mJ

25 An uncalibrated spring balance is found to have a period of oscillation of 0.314 s, when a 1 kg weight is suspended from it, how much does the spring elongate (in cm), in when a 1 kg weight is suspended from it ? Take π = 3.14

Solution

Here, T = 0.314 s ; m = 1 kg

Now, $T = 2\pi\sqrt{\frac{m}{k}}$ or $k = \frac{4\pi^2 m}{T^2}$

or $k = \frac{4\pi^2 \times 1}{(0.314)^2} = \frac{4 \times (3.14)^2 \times 1}{(0.314)^2} = 400 \text{ N m}^{-1}$

When spring is loaded with a weight 1 kg,

m g = k l

or $l = \frac{\text{m g}}{k}$

or $l = \frac{1 \times 9.8}{400} = 0.0245 \text{ m} = 2.45 \text{ cm}$

CHEMISTRY

1 Conductometric titration curve of an equimolar mixture of a HCl and HCN with NaOH(aq) is :

Solution

Molar conductivity of H^+ and OH^- are very high as compare to other ions. Initially, conductance of solution sharply decreases due to consumption of free H^+ then increases due to formation of salt (NaCN) and after complete neutralization, further sharply increases due to presence of OH^-.

2 Which of the following carbides produces propyne on reaction with water ?

Solution

$$\begin{array}{llcl}
\text{(a)} & CaC_2 + H_2O & \rightarrow & Ca(OH)_2 + HC \equiv CH \uparrow \\
\text{(b)} & Be_2C + 2H_2O & \rightarrow & 2BeO + CH_4 \uparrow \\
\text{(c)} & Al_4C_3 + 12H_2O & \rightarrow & 4Al(OH)_3 + 3CH_4 \uparrow \\
\text{(d)} & Mg_2C_3 + 4H_2O & \rightarrow & 2Mg(OH)_2 + H_3C - C \equiv CH \uparrow
\end{array}$$

3 An organic compound contains 49.3% carbon, 6.84% hydrogen and its vapour density is 73. What is the molecular formula of the compound ?

Solution

% of oxygen = 100 - (49.3 + 6.84)

= 43.84 %

Element	%	At. wt.	$\frac{\text{Percentage}}{\text{at. wt}}$	Simplest ratio
C	49.30	12	$\frac{49.30}{12} = 4.11$	$\frac{4.11}{2.74} = 1.5 \times 2 = 3$
H	6.84	1	$\frac{6.84}{1} = 6.84$	$\frac{6.84}{2.74} = 2.5 \times 2 = 5$
O	43.84	16	$\frac{43.84}{16} = 2.74$	$\frac{2.74}{2.74} = 1.0 \times 2 = 2$

Hence, empirical formula = $C_3H_5O_2$

Empirical formula weight

= 12 x 3 + 5 x 1 + 16 x 2

= 73

Molecular weight = 2 x vapour density

= 2 X 73

= 164

$\therefore \quad n = \frac{\text{molecular weight}}{\text{empirical formula weight}}$

$= \frac{146}{73}$

= 2

$\therefore$ Molecular formula of compound

= $(C_3H_5O_2)_2$

= $C_6H_{10}O_4$

4 The temperature, at which a gas shows maximum ideal behaviour, is known as

Solution

Boyles' temperature is the temperature at which a real gas exhibit ideal behaviour for considerable range of pressure. It is related with van der Waals' constant as

$T_B = \frac{a}{bR}$

5 Only Iodine forms Hepta-fluoride IF_7, but Chlorine and Bromine give Penta-fluorides. The reason for this is :

Solution

Due to larger size of iodine atom, it can accommodate upto seven small fluorine atoms around, while due to smaller sizes of chlorine and bromine atoms cannot accommodate seven fluorine atoms, i.e., steric factor dominate in case of chlorine and bromine.

6 The following data were obtained from the first order decomposition of $2\ \mathrm{A}\ (\mathrm{g}) \rightarrow \mathrm{B}\ (\mathrm{g}) + \mathrm{C}(\mathrm{S})$ at a constant volume and at a particular temperature

S No.	Time	Total pressure in Pascal
1	At the end of 10 min	300
2	After completion	200

The rate constant in min^{-1} is

Solution

$$\begin{array}{cccccc} & 2\mathrm{A} & \rightarrow & \mathrm{B} & + & \mathrm{C} \\ \mathrm{t}=0 & 2\mathrm{P} & & 0 & & - \\ \text{After 10 min} & 2\mathrm{P}-2\mathrm{x} & & \mathrm{x} & & - \\ \text{After completion} & 0 & & \mathrm{P} & & - \end{array}$$

Given, P = 200 {after the completion}

$2\mathrm{P}-\mathrm{x}=300$ (at the end of 10 min)

$\mathrm{x}\ = 100$

$\mathrm{k}=\frac{2.303}{10} \log \frac{200}{100}=0.0693\ \mathrm{min}^{-1}$

7 Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, Which of the following statements is incorrect ?

Solution

Ln^{3+} compounds are generally coloured in the solid state as well as in aqueous soloution.colour appears due to presence of unpaired *f*-electrons which undergo *f-f* transition.

8 s-electrons of the valence shell of some elements show reluctance in bond formation. Such elements are ..... and belong to ...... ;

Solution

Heavy p-Block elements of group III_A, IV_A, V_A and in periods No.5 & No.6 with configuration $ns^2\ np^{1-3}$. The ns electrons due to close proximity to nucleus do not participate in electrovalency, only p-electrons do so.

9 A new Iron containing compound can have either of the two possible formulae $K_3\left[Fe(C_2O_4)_3\right]$ or $K_2\left[Fe(C_2O_4)_2\cdot(H_2O)_2\right]\cdot$ A 1.356 g of compound in acid is dissolved which converts oxalate to oxalic acid the solution required 34.5 mL of 0.108M $KMnO_4$ to reach point of equivalence. Which is correct formula of compound. Report its molar mass

Solution

Meqs of $KMnO_4$ = 34.5 x 0.108 x 5 = 18.63 Meqs of compound

Equivalent weight of compound $= \frac{1.356}{18.63}\times 1000 = 72.78$

Redox changes are

$$\begin{aligned}\left[Fe(C_2O_4)_3\right]^{3-} &\dashrightarrow Fe^{3+} + 6CO_2 + 6e^-\\ \left[Fe(C_2O_4)_2(H_2O)_2\right]^{2-} &\dashrightarrow Fe^{3+} + 4CO_2 + 5e^-\end{aligned}$$

Equivalent weights are

$$\begin{aligned}\text{for}\quad K_3\left[Fe(C_2O_4)_3\right] &= \tfrac{437}{6} = 72.8\\ \text{for}\quad K_2\left[Fe(C_2O_4)_2(H_2O)_2\right] &= \tfrac{346}{5} = 69.2\end{aligned}$$

i.e., compound is $K_3\ [Fe(C_2O_4)_3]$ and its molar mass = 437 g/mole.

10 Which among the following statements is false ?

Solution

(i) Osmotic pressure $\pi = i\ CRT$

$\therefore$ concentration is same so $\pi \propto i$ (number of ions)

i for $BaCl_2 = 3$, $KCl = 2$, $CH_3COOH\ \le 1 < i \le 2$ and for sucrose $= 1$

So, order of osmotic pressure $BaCl_2 > KCl > CH_3COOH > Sucrose$
Acetic acid does not dissociate completely. Hence, its van't Hoff factor will be in between 1 and 2.

(ii) The extent of depression in freezing point varies with the number of solute particles for a fixed solvent only and it's a characteristic feature of the nature of solvent also $\Delta T_f = k_f \times m$
For different solvents,value of k_f is also different so, for two different solvents the extent of depression may vary even if same number of solute particles be dissolved in them.

11 ${}_{90}Th^{228}$ emits four alpha and one beta particle. Number of neutrons in daughter element is

Solution

$$_{90}Th^{228} \xrightarrow[-4\alpha]{} {}_{82}Th^{212} \xrightarrow[-\beta]{} \underset{\text{Daughter element}}{{}_{83}Th^{212}}$$

Number of neutrons = Mass number - Atomic number

$= 212 - 83 = 129$

12 What volume of 1.00 mol L^{-1} aqueous sodium hydroxide is neutralized by 200 mL of 2.00 mol L^{-1} aqueous hydrochloric acid? Find the mass of sodium chloride produced. The Neutralization reaction is $NaOH\,(aq.) + HCl\,(aq.) \rightarrow NaCl(aq.) + H_2O(l)$.

Solution

$$\text{Meq. of NaOH} = \text{Meq. of HCl} = \text{Meq. of NaCl}$$

$$M \times V_{mL} = M \times V_{mL}\ (N_{HCl} = M_{HCl}\ ;\ N_{NaOH} = N_{NaOH})$$

$$1 \times V = 2 \times 200$$

$$\therefore \quad V = 400 \text{ mL}$$

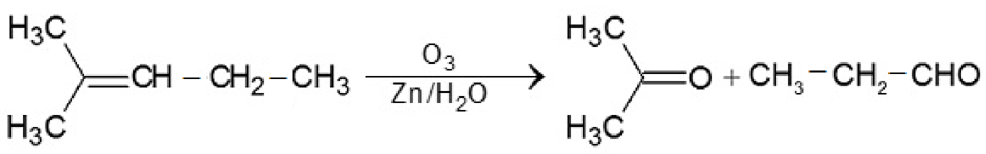
$$\text{Millimoles. of NaCl} = 2 \times 200 = 400$$

$$\therefore \quad \frac{w}{58.3} \times 1000 = 400$$

$$\therefore \quad w_{NaCl} = 23.4 \text{ g}$$

13 2-Methylpent-2-ene on reductive ozonlysis will give

Solution



14 Which one of the following constitutes a group of the isoelectronic species?

Solution

Species having the same number of electrons called as isoelectronic species. By calculating the number of electrons in each species given here, we get

$CN^{-}\ (6+7+1=14); N_2\ (7+7=14);$

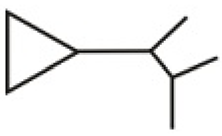
$O_2^{2-}\ (8+8+2=18); C_2^{2-}\ (6+6+2=14);$

$O_2^{-}\ (8+8+1=17)\ ; NO^{+}\ (7+8-1=14)$

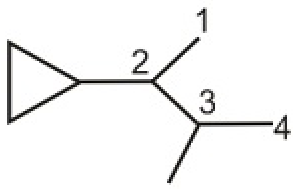
$CO\ (6+8=14); NO\ (7+8=15)$

From the above calculation we find that all the species listed in choice c have 14 electrons each so it is the correct answer.

15 The correct IUPAC name of the following compounds is



Solution



2-cyclopropyl-3-methylbutane.
Longest chain is the open chain not the cyclic one. Numbering is done in alphabetical order of locants since they are equidistant from the terminals.

16 Which one of the following is manufactured by the electrolysis of fused sodium chloride

Solution

$\underset{\text{(fused)}}{2NaCl} \rightarrow 2Na^{+} + 2Cl^{-}$

Anode: $2Cl^{-} \rightarrow 2e^{-} + Cl_2$ (oxidation)

Cathode: $2Na^{+} + 2e^{-} \rightarrow 2Na$ - (reduction)

17 The solubility product of AgCl is 1.8×10^{-10}. Precipitation of AgCl will occur only when equal volumes of which of the following solutions are mixed?

Solution

After mixing,
$\left[\mathrm{Ag}^{+}\right] = \frac{1}{2} \times 10^{-4} = 5 \times 10^{-5}\,\mathrm{M}$
$[\mathrm{Cl}^{-}] = \frac{1}{2} \times 10^{-4} = 5 \times 10^{-5}\,\mathrm{M}$
$\mathrm{Q_{sp}} = \left[\mathrm{Ag}^{+}\right]\,[\mathrm{Cl}^{-}] = \left(5 \times 10^{-5}\right)^{2} = 2.5 \times 10^{-9}$
Since, $\mathrm{Q_{sp}} > \mathrm{K_{sp}} \rightarrow$ precipition takes place

18 A body centred cubic lattice is made up of hollow sphere of B. Sphere of solid A are present in hollow sphere of B. Radius of A is half of the radius of B. What is the ratio of total volume of sphere B unoccupied by A in unit cell and volume of unit cell?

Solution

Effective number of atoms of B present in a unit cell = 2

Total volume of B unoccupied by A in a unit cell

$= 2 \times \frac{4}{3}\left(\mathrm{R}^{3} - \mathrm{r}^{3}\right) \times \pi$

$= \frac{7}{3}\pi\mathrm{R}^{3}\left(\mathrm{r} = \frac{\mathrm{R}}{2}\right)$

Volume of unit cell = a^3

$\left(\frac{4\mathrm{R}}{\sqrt{3}}\right)^{3} = \frac{64}{3\sqrt{3}}\mathrm{R}^{3}\left(\sqrt{3}\mathrm{a} = 4\mathrm{R}\right)$

Ratio of total volume of sphere B unoccupied by A in unit cell and volume of unit cell $= \frac{7/3\pi\mathrm{R}^{3}}{\frac{64}{3\sqrt{3}}\mathrm{R}^{3}} = \frac{7\pi\sqrt{3}}{64}$

19 For an ideal gas $\frac{\mathrm{C_{p,m}}}{\mathrm{C_{v,m}}} = \gamma$. The molecular mass of the gas is M, its specific heat capacity at constant volume is :

Solution

$\because \frac{\mathrm{C_{p,m}}}{\mathrm{C_{v,m}}} = \gamma$ and $\mathrm{C_{p,m}} - \mathrm{C_{v,m}} = \mathrm{R}$

$\therefore C_{v,m} = \frac{R}{\gamma - 1}$

$C_{v,m} = \frac{C_v}{n}$ and $C_v = m.c_v$

$\therefore \frac{R}{\gamma - 1} = \frac{m.c_v}{m} \times M$ and

$\therefore C_v = \frac{R}{(\gamma - 1)M}$

20 A sample of pure PCl_5 was introduced into an evacuated vessel at 473 K. After equilibrium was attained, concentration of PCl_5 was found to be 0.05 mol L^{-1}. If value of Kc is 8.3×10^{-3}, what are the concentration of PCl_3 and Cl_2 at equilibrium?

$PCl_5\ (g) \rightleftharpoons PCl_3\ (g) + Cl_2\ (g)$

Solution

$PCl_5\ (g) \rightleftharpoons PCl_3\ (g) + Cl_2\ (g)$

Given, $[PCl_5]_{equili.}$ = 0.05 mol L^{-1}

Kc = 8.3×10^{-3}

	$PCl_5(g) \rightleftharpoons$	$PCl_3(g)$	$+ Cl_2(g)$
At equilibrium	0.05	x	x

$K_c = 8.3 \times 10^{-3} = \frac{[PCl_3][Cl_2]}{[PCl_5]}$

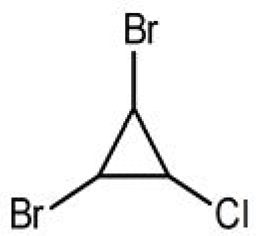
$8.3 \times 10^{-3} = \frac{x^2}{0.05}$

$x^2 = 0.415 \times 10^{-3} = 4.15 \times 10^{-4}$

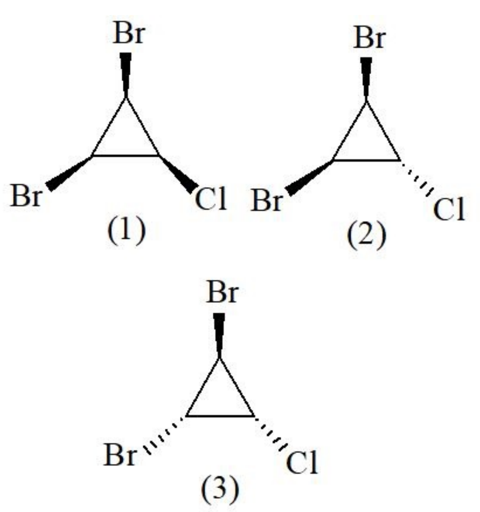
$x = 2.037 \times 10^{-2} = 2.04 \times 10^{-2}$ mol L^{-1}

Hence, $[PCl_3] = [Cl_2] = 2.04 \times 10^{-2}$ mol L^{-1}.

21 Number of stereoisomers possible for the following compound is



Solution



1, 2 have plane of symmetry and are optically inactive. 3 is optically active as there is no plane of symmetry.

22 In how many of the following reactions, one of the products obtained is a yellow coloured precipitate?

(i) $Pb^{2+} + KI \longrightarrow$

(ii) $Pb^{2+} + H_2SO_4 \longrightarrow$

(iii) $Al^{3+} + NaOH \longrightarrow$

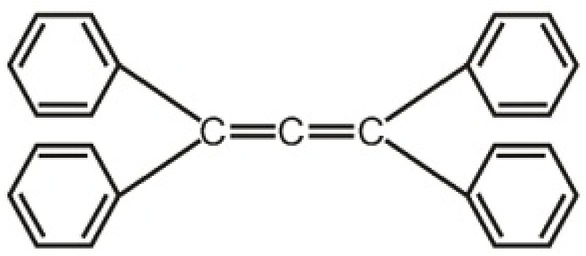
(iv) $Mg^{2+} + Na_2HPO_4 \longrightarrow$

(v) $Pb^{2+} + K_2CrO_4 \longrightarrow$
(vi) $I^- + AgNO_3 \longrightarrow$
(vii) $SO_4^{2-} + BaCl_2 \longrightarrow$

Solution

Reactions (i), (v) and (vi) give yellow ppt. of $PbI_2, PbCrO_4$ and AgI respectively.

23 How many 2° carbon in the following ?



Solution

There are 21 carbon atoms

24 The dipole moment of HBr is 1.6×10^{-30} Coloumb-metre and inter-atomic spacing is $1\ \mathring{A}$. The % ionic character of HBr is

Solution

Charge of electron $= 1.6 \times 10^{-19}\,C$
Dipole moment of $HBr = 1.6 \times 10^{-30}\ C \times m$
Inter-atomic space (distance) = $1\mathring{A}$
$= 1 \times 10^{-10}\ m$
Percentage of ionic character in HBr
$= \frac{\text{Dipole moment of HBr} \times 100}{\text{inter-atomic distance} \times q}$
$= \frac{1.6\times10^{-30}}{1.6\times10^{-19}\times10^{-10}} \times 100$
$= 10^{-30} \times 10^{29} \times 100$
$= 10^{-1} \times 100$
$= 0.1 \times 100$
$= 10\%$

25 How many complexes among the following are paramagnetic
$[Mn(CN)_6]^{3-}$ $[Cr(H_2O)_6]^{3+}$ $[Co(en)_3]^{3+}$
$[V(CO)_6]$, $[Ni(NH_3)_6]^{2+}$ $[Ni(dmg)_2]$
$[Pt(Cl)_2(NH_3)_2]$ $[Cu(NH_3)_4]^{2+}$ $[Cu(CN)_4]^{3-}$

Solution

$[Mn(CN)_6]^{3-}$, $[Cr(H_2O)_6]^{3+}$, $[V(CO)_6]$, $[Ni(NH_3)_6]^{2+}$, $[Cu(NH_3)_4]^{2+}$
are paramagnetic due to the presence of unpaired electrons.

MATHEMATICS

1 If the cube roots of unity are $1, w, w^2$, then the roots of the equation $(x-1)^3 + 8 = 0$ are

Solution

Given equation is $(x-1)^3 + 8 = 0$

$\Rightarrow x = 1 - 2(1)^{1/3}$

$\Rightarrow\ x = 1-2,\ 1-2\omega,\ 1-2\omega^2$ (since $1, w, w^2$ are cube roots of unity)

$\Rightarrow x=-1,\ 1-2w,\ 1-2w^2$

2 $\frac{d}{dx}\left\{\begin{array}{c}\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\\ -\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\end{array}\right\}$ is equal to $\left\{|x|<\sqrt{2}-1\right\}$

Solution

Let $I=\frac{d}{dx}\left\{\begin{array}{c}\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\\ -\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\end{array}\right\}$

Put $x=\tan\theta$ the given equation

$\therefore \quad I=\frac{d}{dx}\left\{\tan^{-1}(\tan 2\theta)+\tan^{-1}(\tan 3\theta)-\tan^{-1}(\tan 4\theta)\right\}$

$=\frac{d}{dx}(\theta)=\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

3 If $\tan\alpha=\left(1+2^{-x}\right)^{-1}, \tan\beta=\left(1+2^{x+1}\right)^{-1},$ then $(\alpha+\beta)$ can be equal to

Solution

We have, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\because \tan\alpha=\frac{1}{1+2^{-x}}$ and $\tan\beta=\frac{1}{1+2^{x+1}}$

$\therefore \tan(\alpha+\beta)=\frac{\frac{1}{1+\frac{1}{2^x}}+\frac{1}{1+2^{x+1}}}{1-\frac{1}{1+\frac{1}{2^x}}\cdot\frac{1}{1+2^{x+1}}}$

$\Rightarrow \tan(\alpha+\beta)=\frac{2^x+2\cdot 2^{2x}+2^x+1}{1+2^x+2\cdot 2^x+2\cdot 2^{2x}-2^x}$

$\Rightarrow \tan(\alpha+\beta)=1$

$\Rightarrow \ \alpha+\beta=\frac{\pi}{4}$

4 If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, then

Solution

Given, $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$

Taking tan on both sides

$\left(\frac{x+y}{1-xy}\right)=1$

$\Rightarrow \quad \frac{x+y}{1-xy}=1$

$\Rightarrow \quad x+y+xy=1$

5 Let two numbers have an arithmetic mean 9 and geometric mean $4,$ then these numbers are the roots of the quadratic equation

Solution

Let the two numbers be α, β

$\therefore \frac{\alpha+\beta}{2}=9$ and $\sqrt{\alpha\beta}=4 \Rightarrow \alpha+\beta=18,\ \alpha\beta=16$

$\therefore$ Required equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow\ \ x^2-18x+16=0$

6 $\lim\limits_{x\to 0}\frac{\cos 2\mathrm{x}-1}{\cos \mathrm{x}-1}=$

Solution

$\lim\limits_{x\to 0}\frac{\cos 2x-1}{\cos x-1}=\lim\limits_{x\to 0}\frac{(2\cos^2 x-1)-1}{\cos x-1}$

$=\lim\limits_{x\to 0}\frac{2(\cos^2 x-1)}{\cos x-1}$

$=\lim\limits_{x\to 0}2\left[\frac{(\cos x+1)\,(\cos x-1)}{(\cos x-1)}\right]$

$=2\lim\limits_{x\to 0}(\cos x+1)=2\,(1+1)=4$

7 Area bounded by the curves $\mathrm{y}=\log_e \mathrm{x}$ and $\mathrm{y}=(\log_e \mathrm{x})^2$ is

Solution

Given curves are $\mathrm{y}=\log_e \mathrm{x}$ and $\mathrm{y}=(\log_e \mathrm{x})^2$

Solving $\log_e \mathrm{x}=(\log_e \mathrm{x})^2 \Rightarrow \log_e \mathrm{x}=0,\ 1 \Rightarrow \mathrm{x}=1$ and $\mathrm{x}=\mathrm{e}$

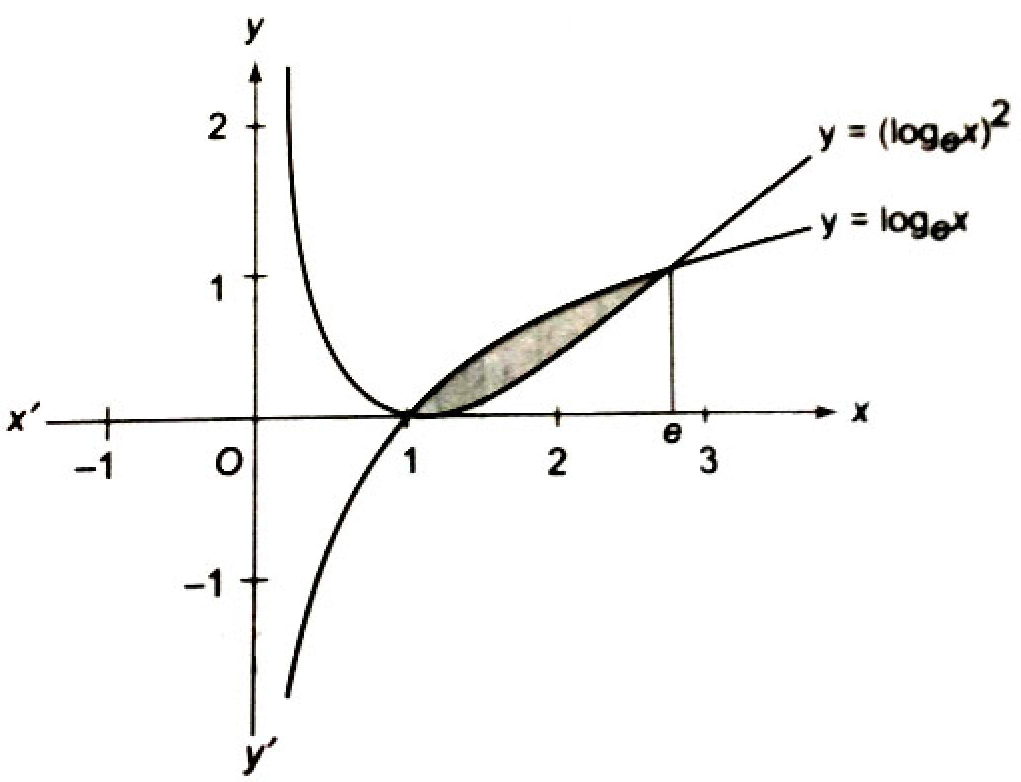
Also, for $1<\mathrm{x}<\mathrm{e},\ 0<\log_e \mathrm{x}<1 \Rightarrow \log_e \mathrm{x}>(\log_e \mathrm{x})^2$

For $\mathrm{x}>\mathrm{e}, \log_e \mathrm{x}<(\log_e \mathrm{x})^2$

$\mathrm{y}=(\log_e \mathrm{x})^2>0$ for all $\mathrm{x}>0$

And when $\mathrm{x}\to 0,\ (\log_e \mathrm{x})^2\to\infty$

From these information, we can plot the graph of the functions.



Then the required area $=\int\limits_1^{\mathrm{e}}\left(\log \mathrm{x}-(\log_e \mathrm{x})^2\right)\mathrm{dx}$

$= \int_1^e \log x dx - \int_1^e (\log_e x)^2 dx$

$= [x \log_e x - x]_1^e - \left[x(\log_e x)^2\right]_1^e + \int_1^e \frac{2\log_e x}{x} x dx$

$= 1 - e + 2[x\log_e x - x]_1^e = 3 - e$ sq. units

8 The solution of the differential equation $\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{1}{(1+x^2)^2}$ is

Solution

Given, $\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{1}{(1+x^2)^2}$

$\therefore \text{IF} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$

The complete solution is

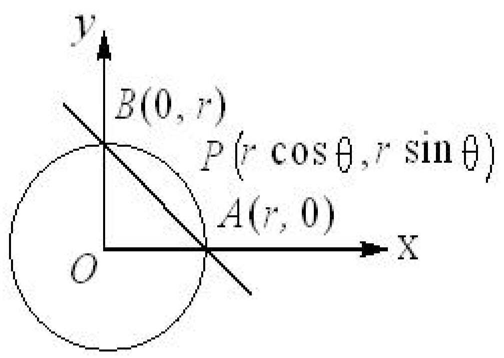
$y\left(1+x^2\right) = \int \left(1+x^2\right) . \frac{1}{(1+x^2)^2} dx + c$

$\Rightarrow y\left(1+x^2\right) = \tan^{-1} x + c$

9 Let AB be a given chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then, the locus of the centroid of the $\Delta\, PAB$ as P moves on the circle is

Solution

Given equation of circle is$x^2 + y^2 = r^2$. Let any point on the circle is $P(r\cos\theta,\ r\sin\theta)$ and let the coordinates of centriod of the triangle be $(\alpha,\ \beta)$



Then, $\alpha = \frac{r + r\cos\theta}{3}$

$\Rightarrow\ \frac{r}{3}\cos\theta = \alpha - \frac{r}{3}$

and $\beta = \frac{r + r\sin\theta}{3}$

$\Rightarrow\ \frac{r}{3}\sin\theta = \beta - \frac{r}{3}$

Now, $\left(\alpha - \frac{r}{3}\right)^2 + \left(\beta - \frac{r}{3}\right)^2 = \frac{r^2}{9}$

$\therefore$ The locus is $\left(x - \frac{r}{3}\right)^2 + \left(y - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2$ which is a circle

10 If $\mathrm{A, B, C}$ are the angles of a triangle and $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$, then which of the following is never true for triangle ABC?

Solution

$R_2 \rightarrow\ R_2 -\ R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix}$$

$R_3 \rightarrow\ R_3 -\ R_2$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix}$$

$= \ (\sin A - \sin B)\ \times\ (\sin B - \sin C)\ \times\ (\sin C - \sin A)$

$\Rightarrow A =\ B$ or $B = C$ or $C =\ A$

So Δ is isosceles or equilateral

11 The acute angle of intersection between the curves $y = \sin x$ and $y = \cos x$ is

Solution

Equation of given curves are

$y = \sin x$ ...(i)

and $y = \cos x$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = \frac{\pi}{4}$

$\therefore$ Point of intersection of curves is $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

For $y = \sin x, \frac{dy}{dx} = \cos x$

(say)

For $y = \cos x \quad \Rightarrow\ \frac{dy}{dx} = -\sin x$

(say)

$\therefore\ \ \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}} = \frac{\frac{2}{\sqrt{2}}}{\frac{1}{2}}$

$\Rightarrow\ \ \tan\theta = 2\sqrt{2}\ \ \Rightarrow\ \theta = \tan^{-1}\left(2\sqrt{2}\right)$

12 Let $x,\ y$ and z be the respective sum of the first n terms, the next n terms and the next n terms of a geometric progression, then $x,\ y,\ z$ are in

Solution

Let A be the 1st term and R be the common ratio of the given G.P.

Sum of the first n terms of the G.P. is $x\ = \frac{A(1-R^n)}{1-R}$. The next n terms form a G.P. series of n terms with the first term as the $(n+1)^{th}$ term of the given G.P. is given by $t_{n+1} = AR^n$. Sum of the next n terms of the G.P. is $y\ \Rightarrow\ y\ = \frac{AR^n(1-R^n)}{1-R}$ and

$z = AR^{2n}\frac{(1-R^n)}{1-R} \Rightarrow\ \ y^2 = zx$

$\therefore$ x, y, z are in G.P.

13 If ${}^{18}C_{15}+2\left({}^{18}C_{16}\right)+{}^{17}C_{16}+1={}^{n}C_3$, then n is equal to

Solution

$${}^{18}C_{15}+2\left({}^{18}C_{16}\right)+{}^{17}C_{16}+1={}^{n}C_3$$

$$\Longrightarrow {}^{18}C_{15}+{}^{18}C_{16}+{}^{18}C_{16}+{}^{17}C_{16}+{}^{17}C_{17}={}^{n}C_3$$

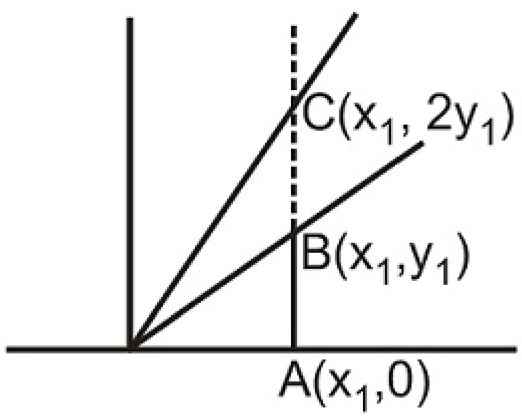
$$\Longrightarrow {}^{19}C_{16}+{}^{18}C_{16}+{}^{18}C_{17}={}^{n}C_3$$

$$\Longrightarrow {}^{19}C_{16}+{}^{19}C_{17}={}^{n}C_3$$

$$\Longrightarrow {}^{20}C_{17}={}^{n}C_3 \Longrightarrow {}^{20}C_3={}^{n}C_3 \Longrightarrow n=20$$

14 Through a point A on the x - axis a straight line is drawn parallel to y - axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If AB = BC then :

Solution



$\mathrm{AB}=\mathrm{BC} \Rightarrow \mathrm{AC}=2\mathrm{AB}.$ Let $\mathrm{AB}=\mathrm{y}$,

$\Rightarrow \mathrm{by}^2+2\mathrm{hx}_1\mathrm{y}+\mathrm{ax}_1^2=0$ has roots $\mathrm{y}_1, 2\mathrm{y}_1$

$3\mathrm{y}_1=-\frac{2\mathrm{hx}_1}{\mathrm{b}}, 2\mathrm{y}_1^2=\frac{\mathrm{ax}_1^2}{\mathrm{b}}$

$\Rightarrow \quad \frac{9\mathrm{y}_1^2}{2\mathrm{y}_1^2}=\frac{4\mathrm{h}^2}{\mathrm{b}^2}\frac{\mathrm{x}_1^2\mathrm{b}}{\mathrm{ax}_1^2}$

$\Rightarrow \quad \frac{9}{2}=\frac{4\mathrm{h}^2}{\mathrm{ab}} \Rightarrow 9\mathrm{ab}=8\mathrm{h}^2$

15 The greatest integer which divides the number $101^{100}-1$ is

Solution

$$\left(1+100\right)^{100}=1+100\ .\ 100+\frac{100.99}{1.2}\left(100\right)^2+\ldots.\Rightarrow 101^{100}-1=100.100\left[1+\frac{100.99}{1.2}+\frac{100.99.98}{3.2.1}100+\ldots..\right]$$

From above it is clear that, $101^{100}-1$ is divisible by $\left(100\right)^2=10000$

16 In an steamer there are stalls for 12 animals and there are horses, cows and calves (not less then 12 each) ready to be shipped. The number of ways, the ship load can be made is ...

Solution

First stall can be filled in 3 ways, second stall can be filled in 3 ways and so on.

$\therefore$ Number of ways of loading steamer

$= 3 \times 3 \times \ldots \times 3\,(12 \text{ times}) = 3^{12}$

17 The domain of the function

$f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

Solution

The function $f(x)$ will be defined, if

$-1 \leq (x-3) \leq 1 \Rightarrow\ 2 \leq x \leq 4$

And $9 - x^2 > 0\ \Rightarrow\ -3 < x < 3$

$\therefore\ \ 2 \leq x < 3$

18 Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\, dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\, dx$. Then, which one of the following is true?

Solution

Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$

as in $x \in (0,1),\ x > \sin x$

$I < \int_0^1 \sqrt{x} dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 \Rightarrow I < \frac{2}{3}$

For, $x \in (0,1),\ \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

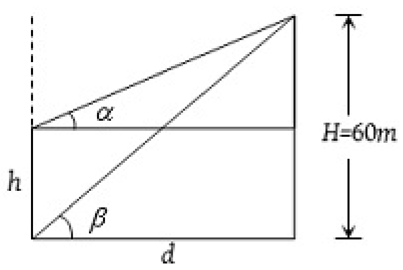
Hence, $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-\frac{1}{2}} dx = 2 \Rightarrow J < 2$

19 From a 60 meter high tower angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60 \sin(\beta-\alpha)}{x}$, then $x =$

Solution

$H = d \tan\beta$ and $H - h = d \tan\alpha$

$\Rightarrow \frac{60}{60-h} = \frac{\tan\beta}{\tan\alpha}\ \Rightarrow\ -h = \frac{60\tan\alpha - 60\tan\beta}{\tan\beta}$



$\Rightarrow h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\,\frac{\sin\beta}{\cos\beta}}\ \Rightarrow x = \cos\alpha\sin\beta.$

20 Out of $3n$ consecutive natural numbers, 3 natural numbers are chosen at random without replacement. The probability that the sum of the chosen numbers is divisible by 3, is

Solution

In $3n$ consecutive natural numbers, either

(i) n numbers are of form $3P$

(ii) n numbers are of form $3P+1$

$(iii)n$ numbers are of form $3P+2$

Here favourable number of cases= Either we can select three numbers from any of the set or we can select one from each set

Hence favourable number of cases
$= {}^nC_3 + {}^nC_3 +^n C_3 + ({}^nC_1 \times {}^nC_1 \times {}^nC_1)$

$= 3\left(\frac{n(n-1)(n-2)}{6}\right) + n^3$

$= \frac{n(n-1(n-2)}{2} + n^3$

Total number of selections $= {}^{3n}C_3$

$\therefore$.Required probability

$= \frac{\frac{n(n-1)(n-2)}{2}+n^3}{\frac{3n(3n-1)(3n-2)}{6}}$

$= \frac{3n^2-3n+2}{(3n-1)(3n-2)}$

Method - II

Let $3n$ consecutive natural numbers are

$1,\ 2,\ 3,\ \ldots .3n$

then let $s_1 = \{1,\ 4,\ 7,\ 10\ \ \ ......\ \ \ 3n-2\}$

$s_2 = \{2,\ 5,\ 8,\ 11,\ \ \ ......\ \ \ 3n-1\}$

$s_3 = \{3,\ 6,\ 9\ \ \ ......\ \ \ 3n\}$

Hence for sum of 3 numbers divisible by 3, we can either select 3 numbers from s_1 or s_2 or s_3 or 1 number from each set
$= n_{c_3} + n_{c_3} + n_{c_3} + n_{c_i} n_{c_i} n_{c_i}$

$= \frac{3.n(n-1)(n-2)}{6} + n^3$

$\frac{n(n^2-3n+2+2n^2)}{2}$

$\frac{n(3n^2-3n+2)}{2}$

and we can select 3 numbers by $3n_{c_3}$ ways.

Hence required probability $= \frac{n(3n^2-3n+2)}{2}$

$\frac{3n(3n-1)(3n-2)}{6}$

$= {}^nC_3 + {}^nC_3 + {}^nC_3 + {}^nC_1.{}^n C_1.{}^n C_1$

21 The area (in sq. units) bounded by the curve $y = \max.\ (x^3,\ x^4)$ and the x-axis from $x=0$ to $x=1$ is

Solution

$\max\left(x^3, x^4\right) = x^3\ \ (\forall x \in (0,1))$
$\therefore$ Required area $= \int_0^1 x^3 dx = \left(\frac{x^4}{4}\right)_0^1 = \frac{1}{4} = 0.25$ sq. units

22 If the value of $(1+tan\ 1^o)\,(1+tan\ 2^o)\,(1+tan\ 3^o)\ldots\ldots\ldots(1+tan\ 44^o)(1+tan\ 45^o)$ is $2^\lambda, \lambda \in N$ then sum of digits of number λ is

Solution

If $A+B=45^o$, then

$\tan(A+B)=1 \because \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

$\Rightarrow \tan A+\tan B=1-\tan A\tan B$

$\Rightarrow (1+\tan A)(1+\tan B\ )=2$

LHS

$=[(1+\tan 1^o).\ (1+\tan 44^o)]\ .\ [(1+\tan 2^o).\ \ (1+\tan 43^o)]\ldots[(1+\tan 45^o)]\ \left[\text{for each } (1+\tan\theta)\left[1+\tan\left(\frac{\pi}{4}-\theta\right)=2\right]\right]$

$=2^{22}.\ (1+1)$

$=2^{23}$

$=2^\lambda$

then $\lambda=23$ hence sum of digits of λ is $2+3$

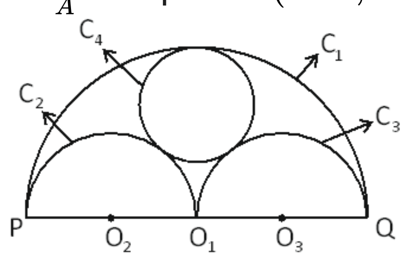
$=5$

23 Let $f(x)=\frac{9x}{25}+c, c>0$. If the curve $y=f^{-1}(x)$ passes through $\left(\frac{1}{4},-\frac{5}{9}\right)$ and $g(x)$ is the antiderivative of $f^{-1}(x)$ such that $g(0)=\frac{5}{2}$, then the value of $[g(1)]$ is, (where $[.]$ represents the greatest integer function)

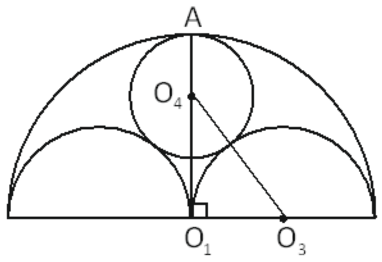
Solution

$y=f(x)=\frac{9x}{25}+c_1$
$\Rightarrow x=\frac{25}{9}(y-c_1)=f^{-1}(y)$
$\Rightarrow f^{-1}(x)=\frac{25}{9}(x-c_1)$
The curve $y=f^{-1}(x)$ passes through $\left(\frac{1}{4},-\frac{5}{9}\right)$
$\Rightarrow -\frac{5}{9}=\frac{25}{9}\left(\frac{1}{4}-c_1\right)$
$\Rightarrow c_1=\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$
$\Rightarrow f^{-1}(x)=\frac{25}{9}\left(x-\frac{9}{20}\right)\ =\frac{25x}{9}-\frac{5}{4}$
$g(x)=\int f^{-1}(x)dx=\int\left(\frac{25x}{9}-\frac{5}{4}\right)dx$
$=\frac{25}{9}\left(\frac{x^2}{2}\right)-\frac{5x}{4}+c_2$
$g(0)=\frac{5}{2}\Rightarrow c_2=\frac{5}{2}$
$\Rightarrow g(x)=\frac{25}{9}\left(\frac{x^2}{2}\right)-\frac{5x}{4}+\frac{5}{2}$
$\Rightarrow g(1)=\frac{25}{18}-\frac{5}{4}+\frac{5}{2}=\frac{50-45+90}{36}=\frac{95}{36}$
$\Rightarrow\ [g(1)]=2$

24 In the figure $PQ,\ PO_1$ and O_1Q are the diameters of semicircles $C_1,\ C_2$ and C_3 with centres at $O_1,\ O_2$ and O_3 respectively and the circle C_4 touches the semicircles C_1, C_2 and C_3. If $PQ=24$ units and the area of the circle C_4 is A sq. units, then the value of $\frac{8\pi}{A}$ is equal to (here, $PO_1=O_1Q$)



Solution



Let the point of contact of C_4 & C_1 is A, center of C_4 is O_4 & radius is equal to r

$\Rightarrow AO_1 = 12 \Rightarrow O_1O_4 = 12 - r$

Also, $O_4O_3 = r + 6$ and $O_1O_3 = 6$

$\Rightarrow (r+6)^2 = (12-r)^2 + 36$

$\Rightarrow 36r = 144 \Rightarrow r = 4 \Rightarrow A = 16\pi$

$\Rightarrow \frac{8\pi}{A} = \frac{8\pi}{16\pi} = \frac{1}{2} = 0.5$

25 Let f, g and h are differentiable functions. If $\mathrm{f}(0) = 1$; $\mathrm{g}(0) = 2$; $\mathrm{h}(0) = 3$ and the derivatives of their pair wise products at $\mathrm{x} = 0$ are $(\mathrm{fg})'(0) = 6$; $(\mathrm{gh})'(0) = 4$ and $(\mathrm{hf})'(0) = 5$, then the value of $(\mathrm{fgh})'(0)$ is

Solution

Let, $\mathrm{y} = \mathrm{fgh}$

$\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{f'gh} + \mathrm{fg'h} + \mathrm{fgh'}$

$= \frac{1}{2}\left(2\mathrm{f'gh} + 2\mathrm{fg'h} + 2\mathrm{fgh'}\right)$

$= \frac{1}{2}\left(\mathrm{h}\left(\mathrm{f'g} + \mathrm{g'f}\right) + \mathrm{g}\left(\mathrm{f'h} + \mathrm{fh'}\right) + \mathrm{f}\left(\mathrm{g'h} + \mathrm{gh'}\right)\right)$

$= \frac{1}{2}\left[\mathrm{h.(fg)'} + \mathrm{g.(fh)'} + \mathrm{f.(gh)'}\right]$

$(\mathrm{fgh})'(0) = \frac{1}{2}\left[\mathrm{h}(0)(\mathrm{fg})'(0) + \mathrm{g}(0)(\mathrm{fh})'(0) + \mathrm{f}(0)(\mathrm{gh})'(0)\right]$

$= \frac{1}{2}(3 \times 6 + 2 \times 5 + 1 \times 4)$

$= \frac{1}{2}(18 + 10 + 4) = \frac{32}{2} = 16$